Large-scale transverse nonlinear interactions in laser beams; new types of nonlinear waves; onset of "optical turbulence"

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Theoretical and experimental results are reported on space-time instabilities of a new type which are driven by large-scale transverse interactions of light beams in media with a cubic nonlinearity.

1. In this letter we wish to present the results of theoretical and experimental research on a new class of space-time instabilities of light beams in media with a cubic nonlinearity: instabilities which are driven by coherent transverse interactions of large scale (the typical scale is $L_\perp \sim d$, where d is the beam width). Depending on the parameter values of the nonlinear medium and the initial and boundary conditions, such instabilities can lead to the generation of new types of optical fields: steady-state, spatially modulated, rotating and helical waves and an optical turbulence.

2. Research on the space and time instabilities of light waves in media with a cubic nonlinearity has been an active branch of nonlinear optics for more than 20 years now. Although many general results were understood in 1965–67 (see the review in Ref. 1), subsequent studies have added much.

Studies of spatial instabilities have revealed a detailed picture of the interaction of, and competition between, the large-scale and small-scale self-focusing of light.[1-3] A temporal analog of the spatial instabilities of a two-dimensional light beam is the modulational instability.[4]

Work on optical bistability has provided new motivation for research on instabilities. Ikeda *et al.*[5] studied temporal instabilities and a chaos in a nonlinear ring resonator which resulted from the finite retardation time of a wave in a feedback circuit. The instabilities, bistabilities, and chaos which have been observed to date by no means exhaust the multifaceted dynamics of nonlinear optical systems. We are actually talking about simply some temporal "projections" of a long list of strong interactions and self-effects, which play out simultaneously in space and time. How does one arrange conditions for observing such phenomena?

In the present letter we wish to stress the role played by the length scale $L_\perp$ of the nonlinear transverse interactions. In a light beam in free propagation, and in optical resonators with conventional layouts, transverse interactions are of the nature of a diffusion and are essentially small in scale. In Refs. 6 and 7 we proposed and implemented some arrangements with a so-called two-dimensional feedback, in which the length scale $L_\perp$ can reach values on the order of the beam width. Below we report studies of instabilities which arise under these conditions and which give rise to an entire hierarchy of new nonlinear light waves.

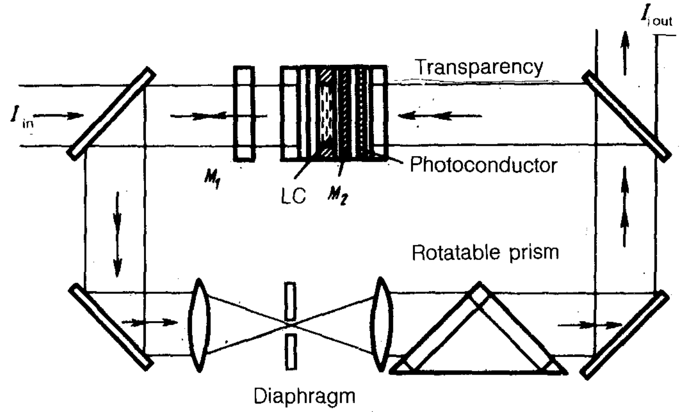

FIG. 1. Arrangement for realizing large-scale transverse interactions: a nonlinear ring resonator with two-dimensional feedback and an optically controllable liquid-crystal phase transparency.[6,7] Mirrors M_1 and M_2 and the liquid-crystal film form a nonlinear interferometer.

3. Figure 1 shows a block diagram of the experimental system in which large-scale transverse interactions are realized by virtue of a two-dimensional feedback. The key elements of this arrangement are an optically controllable phase transparency, in which the phase shift φ depends on the light intensity I and is described by the equation

$$\tau \frac{d\varphi}{dt} + \varphi = \varphi_0 + n_2 klI$$

(l is the thickness, n_2 is the nonlinear increment in the refractive index, and τ is the relaxation time of the nonlinear response); and a rotatable prism, which couples the fields at different points $(\mathbf{r}, \mathbf{r}')$ in the beam cross section.

The nonlinear dynamics of a system of this sort is described by the equation

$$\tau \frac{\partial u(\mathbf{r},t)}{\partial t} + u(\mathbf{r},t) = D\Delta_\perp u(\mathbf{r},t) + K\{1 + \gamma\cos[u(\mathbf{r}',t) + \varphi_0]\}, \qquad (1)$$

where $u = \varphi - \varphi_0$, $K = \chi kln_2 I_{in}$ is the control parameter, χ is the loss parameter, and γ is the visibility of the interference pattern. Transverse interactions are described by the right side of (1): The first term characterizes the small-scale diffusion interactions (the diffusion coefficient D is determined by the nonlocal nature of the response of the transparency and by a combination of diffraction and nonlinear effects), while the second term characterizes large-scale interactions, with $|\mathbf{r} - \mathbf{r}'| \sim d$.

Figure 2 shows some examples of the wave structures and the optical turbulence which are observed experimentally. Parts a and b of Fig. 2 illustrate a rotational instability which arises upon the rotation of the field in the feedback circuit. A change in the rotation angle Δ leads to a change in the velocity and direction of the rotation of the structures. At certain rotation angles Δ_m, the structures remain at rest. During a rotation accompanied by a change in scale, we observe optical helices (Fig. 2, c and

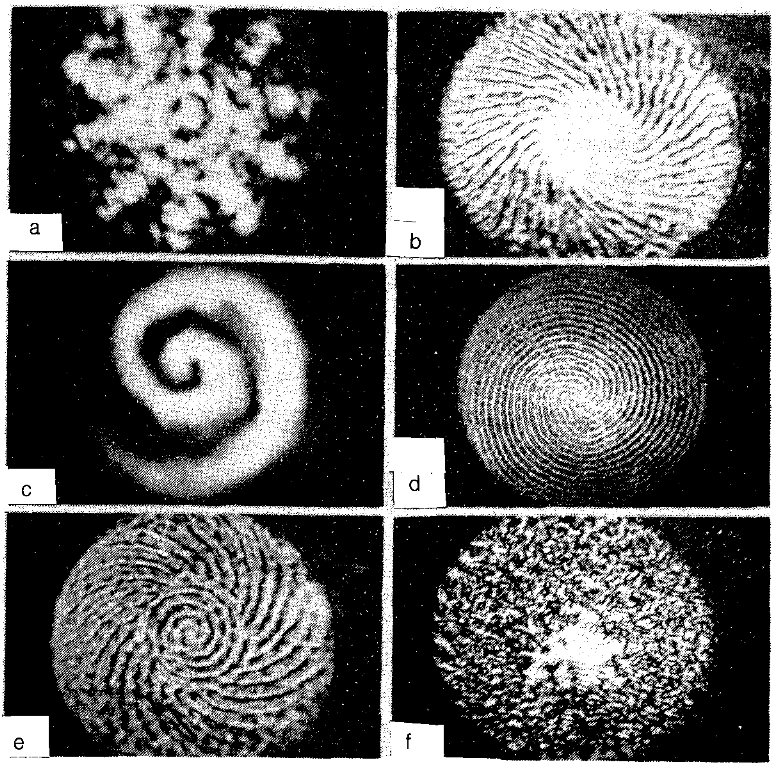

FIG. 2. Nonlinear structures in a resonator with two-dimensional feedback. a,b—Rotating waves; c,d—helical waves (case d corresponds to a lower diffusion coefficient); e—coexistence of structures; f—optical turbulence.

d). Figure 2e illustrates the generation of weakly interacting, coexisting structures. The development of the structures is determined by the control parameter K, the coefficient D, the rotation angle Δ, and the constant phase shift φ_0. An increase in K leads to a stochastic situation and a decay of the structures: to the onset of an optical turbulence (Fig. 2f).

4. We have studied the rotational instability analytically and numerically. We assume for simplicity that the light beam is a thin cone of radius a. For the phase $u(\mathbf{r},t) \equiv u(\theta,t)$ we then have the following equation[1)] in place of (1):

$$\tau \frac{\partial u(\theta,t)}{\partial t} + u(\theta,t) = \frac{D}{a^2} \frac{\partial^2 u(\theta,t)}{\partial \theta^2} + K\{1 + \gamma\cos[u(\theta+\Delta,t) + \varphi_0]\}\,. \tag{2}$$

A perturbation-theory analysis of (2) yields the region of parameter values in which the rotational instability develops, and it also yields the frequency at which the structures rotate:

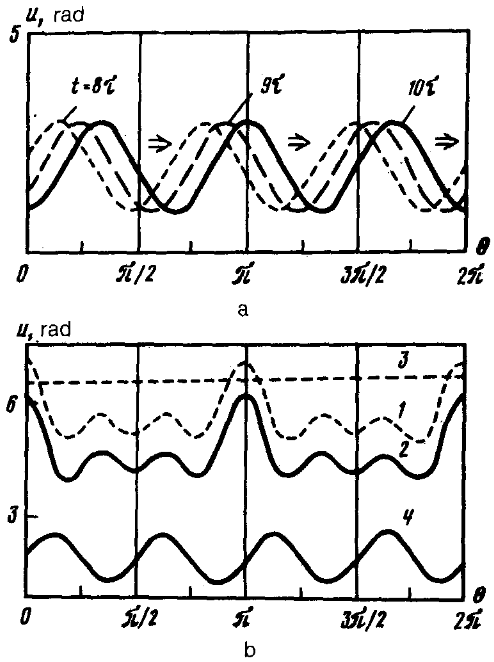

FIG. 3. Results of a numerical simulation of rotating waves. a—Self-similar solution of Eq. (2) with an angular shift $\Delta = 1.9$ rad; b—effect of the initial conditions on the formation of a rotating nonlinear wave. 1,2) Initial phase profiles; 3,4) corresponding wave structures.

$$\omega_n = K\gamma \sin\varphi_{st} \cdot \sin n\Delta / \tau . \qquad (3)$$

Here φ_{st} is a steady-state, spatially uniform solution of Eq. (2). According to (3), immobile, spatially nonuniform structures arise at $\Delta = \Delta_m = (\pi m/n)$ $(m = 0,1,...)$.

Figure 3 shows the results of a numerical solution of Eq. (2). These results agree well with the experimental results. We would like in particular to call attention to the effect of the initial conditions (Fig. 3b): A slight variation of the initial conditions leads to a radical change in the nature of the structures (cf. curves 1,2 and 3,4 in Fig. 3b). It is thus possible to switch structures through a modulation of the optical field which excites the system. The typical switching times $\tau_s \sim \omega_n^{-1}$ can reach 10^{-10}–10^{-11} s for a "fast" Kerr nonlinearity. These circumstances are of fundamental interest to the physics and technology of analog computers.[7]

Self-effects in systems with a two-dimensional feedback result in the generation of fields for which linear optics provides not even remote analogs. Several of the wave structures which we have observed could be associated with the wave structures which are being studied in hydrodynamics, chemistry, and biology (see, for example, the review by Gaponov–Grekhov and Rabinovich[8]). In optics we find some unique opportunities—not available in other fields of nonlinear dynamics—for controlling the scales of longitudinal and transverse interactions, for studying interaction processes, and for controlling structures.

[1)] To the best of our knowledge, equations with a shifted spatial argument of the type in (1) and (2) have not previously been used in nonlinear wave dynamics.

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