Nonstatic objects with a radius less than or equal to the gravitational radius could not exist in the relativistic theory of gravitation

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This paper proves that there is a true singularity on the Schwarzschild sphere for a spherically symmetric solution of the equations of a relativistic theory of gravitation. As a consequence, black holes of the general theory of relativity are absent from the relativistic theory of gravitation, and a gravitational contraction is irreversible.

A relativistic theory of gravitation was proposed in Ref. 1 and developed in Refs. 2 and 3. The theory is based on Minkowski space M_4. Consequently, the initial or basis space for all physical processes, including gravitation, is the space M_4 with the metric γ_{ij}, for which a single map is sufficient for a description (e.g., the Galilean coordinates t, x, y, z). Physical phenomena involving gravitation can be described effectively by a curved space M_4^* which has been allotted a pseudo-Riemannian metric g_{ij}, so that matter interacting gravitationally in space M_4 becomes in a sense free in an effective Riemannian space M_4^*. The effective space (M_4^*, g_{ij}) constructed on Minkowski space in the relativistic theory of gravitation preserves the simple topology of Minkowski space, so the relativistic theory of gravitation does not need to use a set of maps to cover the effective Riemannian space.

In order to distinguish true gravitational effects against the background of coordinate-induced effects in the relativistic theory of gravitation, it is necessary to find the distinction between the space (M_4^*, g_{ij}) and the space (M_4, γ_{ij}). We adopt the complete system of equations of the relativistic theory of gravitation in the conventional form of Hilbert–Einstein equations and a covariant equation which couples g_{ij} and γ_{ij}:

$$R_{ij} - g_{ij}R/2 = 8\pi T_{ij}\ , \tag{1a}$$

$$D_j(\sqrt{-g}\,g^{ij}) = 0. \tag{1b}$$

Here D_j is the covariant derivative with respect to γ_{ij}. A point which deserves particular emphasis is that by virtue of Eqs. (1b) the metric γ_{ij} of Minkowski space appears directly in the equations of the relativistic theory of gravitation and thus influences the description of gravitational processes. Equations (1b) are not a gauge condition, since they have a substantial influence on the gravitational physics of the relativistic theory of gravitation (see, for example, Refs. 2 and 3). Equations (1b) have no bearing of any sort on the choice of a coordinate system. The coordinate system in the relativistic theory of gravitation is specified by the form chosen for the metric γ_{ij}, and Eqs. (1b), like (1a), are universal (i.e., they hold for all gravitational processes) and are generally covariant.

The concept of a Minkowski space cannot be introduced in the general theory of relativity. This is a mathematical fact. It is thus meaningless to speak in terms of Minkowski space in the general theory of relativity. Assertions that the relativistic theory of gravitation and the general theory of relativity are equivalent (e.g., Ref. 4), are erroneous. In the present letter we use the example of a gravitational collapse to show that the physical consequences of the relativistic theory of gravity and the general theory of relativity are substantially different.

The ideas of the relativistic theory of gravitation and Eqs. (1) were used in Refs. 2 to study a spherically symmetric collapse in the example of the simplest solution: the Tolman solution. It was shown that there are no objects with radii smaller than or equal to the Schwarzschild radius in a relativistic theory of gravitation. In the present letter we return to this question, and we prove that this conclusion holds in the general case, without appealing to any specific internal solution (such as the Tolman solution for dust without pressure).

According to the equations of the relativistic theory of gravitation in (1), the metric of the effective Riemannian space, g_{ij}, outside a spherically symmetric object is of the form

$$ds^2 = g_{ij}\,dx^i dx^j = \left(1 - \frac{2m}{r}\right)dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 d\omega^2 \qquad (2)$$

with a Minkowski-space metric

$$\gamma_{ij} = \mathrm{diag}\,(1, -1, -(r-m)^2, -(r-m)^2 \sin^2\theta). \qquad (3)$$

The general conclusion that there are neither static nor dynamic (collapsing) objects with a radius smaller than or equal to the gravitational radius ($r_g \equiv 2m$) in the relativistic theory of gravitation can be proved through a simple analysis of an external solution of (2) and (3). The proof is as follows: In the relativistic theory of gravitation, in contrast with the general theory of relativity, the singularity on the Schwarzschild sphere is a true singularity, not a coordinate singularity. Specifically, the metric g_{ij} given by (2) can be rewritten in the so-called Finkelstein coordinates (τ, R) (see, for example, §102 of Ref. 5),

$$\tau = t + \int dr\,\frac{\sqrt{r_g/r}}{(1 - r_g/r)}\,; \qquad R = t + \int dr\,\frac{\sqrt{r/r_g}}{(1 - r_g/r)}\,, \qquad (4)$$

in such a way that the metric is, by virtue of tensor analysis rules,

$$\bar{g}_{ij}(y) = (\partial x^p/\partial y^i)(\partial x^q/\partial y^j)\; g_{pq}(x)\,,$$

where

$(x^i) = (t, r, \theta, \varphi)$, $(y^i) = (\tau, R, \theta, \varphi)$ and $\partial t/\partial\tau = -(r/r_g)\cdot(\partial t/\partial R) = (1 - r_g/r)^{-1}$,

$$\frac{\partial r}{\partial \tau} = -\frac{\partial r}{\partial R} = -\sqrt{r_g/r}$$

takes on a time-dependent form which is convenient for the analysis of a collapsing object[5]:

$$\bar{g}_{ij} = \mathrm{diag}\,(1, -r_g/r, -r^2, -r^2\sin^2\theta), \tag{5}$$

where

$$r = (r_g)^{1/3}\,[3(R-\tau)/2]^{2/3} .$$

It follows[5,6] from form (5) of the metric $\bar{g}_{ij}$ in the general theory of relativity that there is no true singularity on the Schwarzschild sphere, since coordinate transformations (4) remove this singularity in (5). We are then led to the representation of black holes as objects which are self-closing inside the Schwarzschild sphere and which are inaccessible to observation from the exterior, to the possibility that nonstatic objects could have a radius smaller than r_g, to the irreversibility of gravitational collapse, etc. In the relativistic theory of gravitation, the situation is radically different, since the metric of the plane space, γ_{ij}, appears in an unremovable fashion in the equations of the theory, (1). Accordingly, by removing the singularity in the metric g_{ij} by means of a coordinate transformation we work it into the transformed metric of the plane space. Specifically, applying transformations (4) to metric γ_{ij} in (3), we find the metric $\bar{\gamma}_{ij}$, which is singular at $r = r_g$:

$$\bar{\gamma}_{00} = (1 - r_g/r)^{-2} - r_g/r, \qquad \bar{\gamma}_{11} = (r_g/r)^2(1 - r_g/r)^{-2} - r_g/r,$$
$$\bar{\gamma}_{01} = r_g/r - (r_g/r)(1 - r_g/r)^{-2}, \quad \bar{\gamma}_{22} = \gamma_{22}, \ \bar{\gamma}_{33} = \gamma_{33} . \tag{6}$$

This fact—this unremovability of the singularity in the relativistic theory of gravitation—can be seen particularly clearly in a consideration of the invariant $I_1 = g^{ik}\gamma_{ik}$. This invariant owes its existence to the presence in the relativistic theory of gravitation of both an effective space with a metric g_{ik} and Minkowski space with the metric γ_{ik}, which is not present in the general theory of relativity. Accordingly, there simply are no invariants of the type I_1 in the general theory of relativity. A calculation of the invariant I_1 for solution (2), (3) yields

$$I_1 = \frac{1}{1 - r_g/r} + 1 - r_g/r + 2\left(\frac{r-m}{r}\right)^2 , \tag{7}$$

For the transformed coordinates, I_1 does not change, as can be verified easily through a direct substitution of metrics $\bar{g}_{ij}$ from (5) and $\bar{\gamma}_{ij}$ from (6) in the coordinates (τ, R) in (4). It follows from (6) that the invariant I_1 is singular at $r = r_g$. In a similar way, it can be shown that the invariants $I_2 = g_{ik}\gamma^{ik}$ and $I_3 = R_{ikpq}\gamma^{ip}\gamma^{kq}$ are also singular on the Schwarzschild sphere for solution (2), (3). The Schwarzschild surface in the relativistic theory of gravitation thus corresponds to a true singularity, which cannot be removed through an appropriate choice of coordinate system.

Since a test particle (or light, or the surface of a collapsing object) will asymptotically approach this singularity over an infinite time t of Minkowski space, the relativistic theory of gravitation thus tells us that nature has no spherically symmetric objects, static or nonstatic, with a radius less than or equal to the gravitational radius. Consequently, the black holes of the general theory of relativity are absent from the relativistic theory of gravitation, so there can be no catastrophic collapse of matter.

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