Cancellation of non-Abelian anomalies; the electric charge of quarks

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The condition that non-Abelian anomalies cancel out fixes the electric charge of quarks and determines the width of the decay $\pi^0 \to 2\gamma$.

It is usually assumed that the most convincing evidence that the number (N_C) of colors of quarks whose interaction in locally $SU(N_C)$ symmetric is three comes from, on the one hand, the satisfactory agreement between the theoretical and measured widths of the decay $\pi^0 \to 2\gamma$ and, on the other, the satisfaction of the condition that non-Abelian anomalies cancel out in the dynamic currents of the theory of the electroweak interaction. Without this cancellation, it would be impossible to carry out a self-consistent analysis of a local $SU(2) \otimes U(1)$ symmetry of the electroweak interaction.[1,2]

Nevertheless, it is not difficult to show that these facts would be the same for $N_C \neq 3$. This is the purpose of the present letter. Specifically, we consider a locally $SU(N_C) \otimes SU(2) \otimes U(1)$ symmetric model of the interaction of matter fields (for simplicity, we consider only the first generation of leptons and quarks) and gauge bosons. We now adopt the usual assumptions of the standard $SU(3)_C \otimes SU(2) \otimes U(1)$ theory; The quarks transform in accordance with the $SU(N_C)$ fundamental representation, while the leptons and scalar fields are color singlets. Under the $SU(2) \otimes U(1)$ flavor group, the left-hand components of fermions form doublets, while the right-hand components form singlets:

$$\begin{pmatrix} \nu \\ e \end{pmatrix}_L, \quad \nu_R, \; e_R, \; \begin{pmatrix} u \\ d \end{pmatrix}_L, \; u_R, \; d_R \quad .$$

Scalar fields also form a doublet. By formally combining the right-hand components of leptons and quarks in $SU(2)_R$ doublets, we can write the electric charges of the particles in the form $Q = \tau_3/2 + y$, where τ_3 is the Pauli matrix, and y is a hypercharge, so that $y = \langle Q \rangle$ is the expectation value of the electric charge of the multiplet. It follows that the hypercharge of leptons is $y_l = -1/2$, and we find the hypercharge of quarks, y_q, from the condition that the non-Abelian anomalies cancel out in the dynamic currents. In the last point we have thus deviated from a postulate of the standard $SU(3)_C \otimes SU(2) \otimes U(1)$ theory, in which the value of the hypercharge y_q is found from the hypothesized magnitude of the electric charges of the u and d quarks, whose values are in turn fixed by the $SU(3)$ flavor symmetry in a nonrelativistic quark model.

The presence of anomalies in the fermion current J_μ and J^a_μ, $a = 1, 2, 3$, which interact with the gauge fields B_μ and W^a_μ means that the divergence and the covariant derivative of the currents are not zero at the single-loop level and are given by (we are using the definition of hypercharge given above)[3]

$$\partial_\mu J^\mu + \ldots = -\frac{y}{24\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\,\partial_\mu\left(\mathrm{tr}\left(W_\nu \partial_\rho W_\sigma + \frac{i}{2} W_\nu W_\rho W_\sigma\right) - \frac{3}{2} B_\nu \partial_\rho B_\sigma\right),$$

$$(D_\mu J^\mu)_a + \ldots = -\frac{y}{24\pi^2}\,\epsilon^{\mu\nu\rho\sigma}\,\partial_\mu\,\mathrm{tr}\,\tau_a\left(B_\nu \partial_\rho W_\sigma + \frac{i}{4} B_\nu W_\rho W_\sigma\right).$$

Here the ellipsis (...) means terms representing the Yukawa interaction of fermions and Higgs fields. The condition for the cancellation of the anomalies thus takes the form

$$N_C y_q + y_l = 0 \quad \text{or} \quad N_C \langle Q_q \rangle + \langle Q_l \rangle = 0 \; .$$

It follows that the condition for the cancellation of non-Abelian anomalies in the dynamic currents of the theory of the electroweak interaction fixes the hypercharge of the quarks, $y_q = 1/(2N_C)$, and thus the electric charge of the quarks;

$$Q_u = (1+N_C)/(2N_C), \qquad Q_d = (1-N_C)/(2N_C).$$

If $N_C = 3$, we have the standard $SU(3)_C \otimes SU(2) \otimes U(1)$ theory, with $y_q = 1/6$ and $Q_u = 2/3$, $Q_d = -1/3$.

It is now a straightforward matter, using, for example, the effective chiral Langrangian of pseudoscalar mesons with a Wess–Zumino term,[4] to verify that the anomalous vertices of the interactions of pions with photons ($\gamma\gamma\pi^0$, $\gamma\pi^+\pi^-\pi^0$, $\gamma\gamma 3\pi$) are proportional to $N_C y_q$, i.e., are determined by the condition for the cancellation of non-Abelian anomalies: $N_C y_q = 1/2$. It is thus clear that the width of the decay $\pi^0 \to 2\gamma$ is not a suitable test for determining N_C.

Another quantity which provides evidence for $N_C = 3$ is the total cross section for the process $e^+e^- \to$hadrons. A naive calculation of the cross-section ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-) = N_C \Sigma Q_q^2$ leads to a satisfactory agreement with experimental data. Nevertheless, the experimental observation of chromodynamic-coherence effects has refuted the model of independent fragmentation of jets,[5] whose validity was based on the derivation of this value of R. Furthermore, it is not difficult to verify that in the leading $1/N_C$ expansion the matrix element for the process $e^+e^- \to$hadrons is proportional to $e^2 N_C Q_q$, and to calculate the total cross section, we need to deal with interference effects correctly.

A soliton solution of the effective chiral Langrangian with a Wess–Zumino term can be regarded as a baryon, as we know. The Wess–Zumino term assigns soliton quantum numbers which correspond to the quantum numbers of the observed baryons. The value of N_C must be odd in this case.[4,6] In the $SU(N_C) \otimes SU(2) \otimes U(1)$ model which we are considering here, it is again a straightforward matter to generate a microscopic description of baryons (a similar picture arose in a study of baryons in the $1/N_C$ expansion[7]). We assume $N_C = 2k + 1$. The proton may then consist of $(k+1)$ u quarks and k-d quarks, and a neutron of k-u quarks and $(k+1)$ d quarks. Since the spin-spin forces between u and d quarks are of such a nature that an antiparallel configuration is favored from the energy standpoint, the lowest state of such a system will have a spin of 1/2. The state of the Δ^{++} isobar can now be represented as consisting of $(k+2)$ u quarks and $(k-1)$ d quarks. In general, if $N_C \neq 3$, baryons with isospin up to $N_C/2$ must exist, as we know.[4] Such states have been sought for a long time now, but there is still no reliable interpretation of the experimental data.

Superstring theories are presently the most popular schemes for combining all interactions.[8] From this standpoint it is clear that N_C cannot be very large. Furthermore, it is interesting to consider as candidates for the color symmetry group not only $SU(N_C)$ but also other Lie groups.[1)] An analysis of this sort was carried out partially by Witten,[4] who considered, along with $SU(N_C)$, the $O(N_C)$ and $Sp(N_C)$ groups. If quark fields transform under a fundamental representation of the color group, and if the hadrons are color singlets, then a necessary condition for the existence of baryons is that the fundamental representation be complex, as Witten has shown.[4] We know that only two Lie groups satisfy this condition: $SU(N_C)$ and E_6. Accordingly, the exceptional E_6 group is also a possible candidate for the role of the color gauge group.

In summary, we have found that (first), the width of the decay $\pi^0 \to 2\gamma$ is not a suitable test for determining the number of quark colors and (second), the condition for the cancellation of the non-Abelian anomalies in the dynamic currents of the electroweak interaction fixes the electric charge of the quarks.

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