Classical plane-wave states of a chiral pion field and collisions of ultrahigh-energy nuclei

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Exact plane-wave solutions are derived for the equations of motion of a classical pion field described by a chiral Lagrangian. A possible relationship between these solutions and experiments on collisions on high-energy ions is discussed.

1. Recent experiments on collisions of high-energy nuclei have been using energy densities on the order of several gigaelectron volts per cubic femtometer in a volume on the order of tens or even hundreds of cubic femtometers.[1] Hundreds of pions are produced in the process. Measurements of two-particle pion correlations seem to indicate that the production of pions from a fairly large volume (with linear dimensions up to ~8 fm) occurs coherently (details below). The situation suggests that in a certain stage of the process a classical pion field can form in a certain finite volume, and the decay of this field in a coherent fashion results in the production of π mesons. In the present letter we show that in a theory with a chiral Lagrangian there are classical plane-wave solutions which correspond to an energy density on the order of, say, 200 MeV/fm^3, i.e., to an energy density completely attainable in experiments today. A classical pion field of this sort, which exists a finite time interval in a finite volume, could serve as a coherent source of the emission of pions with fairly large dimensions. Actually, it would be difficult to even imagine another coherent source of pion production with linear dimensions above a few fermis (femtometers).

2. We write the chiral Lagrangian of the pion field in the parametrization $\hat{\phi} = \phi_4 + i(\tau_k \phi_k)$, where it takes the form of the Lagrangian of an n-field $(\phi_a = f_\pi n_a)$:

$$\mathcal{L} = f_\pi^2/_2 (\partial_\mu n_a)(\partial_\mu n_a), \quad n_a^2 = 1, \quad a = 1, 2, 3, 4, \quad i = 1, 2, 3, \tag{1}$$

The equations of motion are found by varying the function $f_\pi^{-2}\mathcal{L} - \frac{1}{2}\lambda(n_a^2 - 1)$, where λ is an undetermined Lagrange multiplier:

$$\partial^2 n_a + \lambda n_a = 0. \tag{2}$$

Finding λ from the condition $n_a^2 = 1$, we find a nonlinear equation for n_a

$$\partial^2 n_a - n_a(n_b \partial^2 n_b) = 0, \quad \lambda = -(n_b \partial^2 n_b). \tag{3}$$

Let us attempt to find a class of solutions of these equations in the form

$$n_a = A_a \cos(\omega t - \mathbf{k}\mathbf{r} + \varphi_a), \quad \omega^2 - \mathbf{k}^2 = m^2, \tag{4}$$

for which λ is a constant:

$$\lambda = -(n_a \partial^2 n_a) = m^2 . \tag{5}$$

Since we have, according to (4),

$$\lambda = m^2 \sum_{a=1}^{4} A_a^2 \cos^2(\omega t - \mathbf{kr} + \varphi_a) , \tag{6}$$

to ensure the satisfaction of (5) it is sufficient to require

$$\sum_{a=1}^{4} A_a^2 \sin 2\varphi_a = 0, \qquad \sum_{a=1}^{4} A_a^2 \cos 2\varphi_a = 0, \qquad \sum_{a=1}^{4} A_a^2 = 2. \tag{7}$$

If conditions (7) are imposed on the constants A_a and φ_a, plane waves (4) are indeed solutions of nonlinear equations (3).

Let us examine the complex numbers $z_a = (A_a \exp i\varphi_a)^2$. In this case Eqs. (7) take the form

$$\sum_{a=1}^{4} z_a = 0, \qquad \sum_{a=1}^{4} |z_a| = 2. \tag{8}$$

This result means that the complex numbers z_a, thought of as two-dimensional vectors with an unspecified origin, form a closed quadrangle, and the perimeter of this quadrangle is 2 (Fig. 1).

The class of solutions which has been found here is actually one-dimensional. In this sense, it contains six arbitrary parameters [the nine quantities A_a, φ_a, and m minus three conditions (7)]. The six-parameter family for the three independent functions $n_1 n_2 n_3$ is a general solution of the one-dimensional problem. Let us consider two particular cases.

In the first case we have $n_2/n_1 = \text{const}$; this situation is possible if $\varphi_2 = \varphi_1 + \pi m$, $m = 0,1,2...$, i.e., if $\arg z_2 = \arg z_1 + 2\pi m$. Geometrically in this case, we have the situation shown in Fig. 2. In this particular case there is a rotation of the vector n_3 in a fixed plane in an isotopic three-dimensional space. The length $|\mathbf{n}| = \sqrt{1 - n_4^2}$ also oscillates.[1])

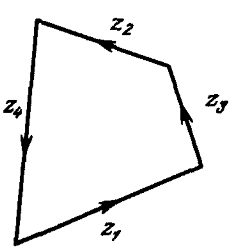

FIG. 1.

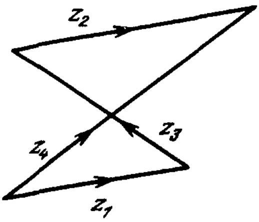

FIG. 2.

In the second case we have $n_4 = 0$. In this case the quadrangle in Fig. 1 degenerates into a triangle. The absolute magnitude of the pion field remains constant, $|\mathbf{n}| = 1\,(|\vec{\phi}| = f_\pi)$, but all three projections oscillate, so the vector $\boldsymbol{\phi}$ executes a complicated motion in isotopic space (a hula-hoop motion).

General solution (4) has an interesting property. It is easy to see that the expression for the energy-momentum tensor does not depend on the coordinates or the time:

$$T_{\mu\nu} = (k_\mu k_\nu - \frac{1}{2} m^2 g_{\mu\nu}) f_n^2 , \qquad m^2 = \omega^2 - \mathbf{k}^2 . \tag{9}$$

In particular, the energy density $H = T_{00}$ takes the form

$$H = \frac{f\pi^2}{2} (\omega^2 + \mathbf{k}^2) . \tag{10}$$

Correspondingly, the isospin-density vector does not depend on the coordinates or the time:

$$j_\mu^i = {}^1\!/_{2i} [(\partial_\mu \phi^* T_i \phi) - \phi^* T_i \partial_\mu \phi]$$

$$= \epsilon_{ikl} \phi_k \; \partial_\mu \phi_l = f\pi^2 k_\mu \; \epsilon_{ikl} A_k A_l \sin(\varphi_k - \varphi_l) .$$

3. It appears that the classical solutions found in the preceding section of this letter may be pertinent to experiments on the collisions of heavy nuclei in the following sense. Let us imagine that over a finite time interval $0 \leqslant t \leqslant T$ (from some instant after the collision to an instant at which the energy density of the pion-field blob which has formed has become small) there is a classical pion field described by Eqs. (4) in a finite volume. Outside the time interval $(0, T)$ and the volume V, the pion field is described by the free d'Alembert's equation. In this crude model, the emission of the pions is determined by a "source" with a density

$$\rho_a(\mathbf{r}, t) = - n_a (n_b \partial^2 n_b) = m^2 A_a \cos(\omega t - \mathbf{kr} + \varphi_a), \tag{11}$$

which is bounded in space and time. The Fourier component $n_a(\mathbf{R}, E)$ of the (massless) pion field which is emitted $(a \neq 4)$ is given in the wave zone by the integral

$$n_a(\mathbf{R},E) = \frac{e^{ipR}}{4\pi R} \int_V e^{-i\mathbf{p}\mathbf{r}} \rho_a(\mathbf{r},E)d^3r, \quad \mathbf{p} = E\frac{\mathbf{R}}{R}, \quad |\mathbf{p}| = E. \tag{12}$$

Substituting ρ_a from (11) into this expression, we easily find the following expression for the amplitude of the production of the pion field:

$$F_a(\mathbf{p},E) = f_a(\mathbf{p},E) + f_a^*(-\mathbf{p},-E),$$

$$f_a(\mathbf{p},E) = \frac{f_\pi m^2 A_a e^{i\varphi_a}}{8\pi i(E+\omega)} (e^{i(E+\omega)T} - 1) \int_V e^{-i(\mathbf{p}+\mathbf{k})\mathbf{r}} d^3r. \tag{13}$$

Here E and $\mathbf{p}$ are the energy and momentum of the emitted pion ($E = |\mathbf{p}|$), and ω and $\mathbf{k}$ characterize the "source," the classical pion field ($E^2 = \mathbf{k}^2 + m^2$). Since our model is very crude, and since we still do not know the parameters of the classical field, expression (13) contains little information. My only purpose in deriving it was to refine just exactly what we have in mind when we talk about the relationship between classical solutions and the production of pions in high-energy nuclear collisions.

4. If the picture drawn above does indeed prevail, two phenomena which would appear to have experimental manifestations could occur. 1) Since any specific solution of (4), (8) is not isotopically symmetric, there can be a significant violation of isotopic invariance in individual collisions: The numbers of neutral and charged pions produced could be different, despite the large total number of pions, i.e., under the condition that the statistical fluctuations are small.[4] 2) Experimentally, one measures the correlation function

$$R = \frac{d^2n/d^3p_1d^3p_2}{(dn/d^3p_1)(du/d^3p_2)}, \tag{14}$$

which is equal to the ratio of the number of events involving the production of two pions with momenta p_1 and p_2 to the single-particle inclusive distributions. It is easy to understand that for the coherent production of pions we would have $R = 1$, while for an incoherent mechanism R would depend on the momentum difference $\Delta p = p_1 - p_2$, $R = R(\Delta p)$; here we would have $R(0) = 2$ and $R(\infty) = 1$ (Ref. 5, for example). Actually, what is measured experimentally is the jump in R at small and large values of Δp. This jump turns out to be less than unity; this result is difficult to understand in the case of a purely incoherent production mechanism, but it would be a completely natural result if a coherent production of pions occurred in several cases.

A separate study is required for a theoretical estimate of the probability for the production of a classical field in a finite volume in a collision of heavy nuclei. We can, however, offer a crude estimate of this probability based on experimental data. Let us assume that the probability for a coherent-production event is w, while that for an incoherent event is, correspondingly, $1 - w$. We can then replace (14) by

$$\frac{d^2n}{d^3p_1d^3p_2} = 1w\left(\frac{dn}{d^3p}\right)^2_{\rm coh} + R(1-w)\left(\frac{dn}{d^3p}\right)^2_{non\text{-}coh}, \tag{15}$$

where dn/d^3p are single-particle inclusive distributions for the coherent and incoherent cases, respectively, and we have $R = 2$ for $\Delta pa \ll 1$ and $R = 1$ for $\Delta pa \gg 1$, where a is the size of the source. For R in the coherent case we inserted the value $R = 1$. (In the functions $dn/d^3p_{1,2}$, we can ignore the circumstance that we have $p_1 \neq p_2$). The ratio of the doubly inclusive spectra in (15) in the cases $\Delta pa \ll 1$ and $\Delta pa \gg 1$ is measured experimentally:

$$1+\lambda \equiv \frac{(d^2n/d^3p_1d^3p_2)_{\Delta p = 0}}{(d^2n/d^3p_1d^3p_2)_{\Delta p = \infty}} = \frac{w\xi + 2(1-w)}{w\xi + (1-w)}, \tag{16}$$

where $\xi = (dn/d^3p)^2_{coh}/(dn/d^3p)^2_{non\text{-}coh}$. Hence

$$w = \frac{1-\lambda}{1-\lambda+\lambda\xi}. \tag{17}$$

Experimentally, λ has been found[2] to be $\lambda \approx 0.77$. The multiplicity in coherent events might be slightly higher than that in incoherent events (see, for example, the "centaur" case, in which there is a pronounced violation of isotopic invariance[4]). With $\xi \approx 1$–10, the probability for coherent production is $w \approx 0.2$–0.03.

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[1)]After general solutions (4) and (8) had been derived, I learned of Ref. 3, where this particular solution was written. It appears at first glance that solution (3.8) of Ref. 3 depends on six parameters ($C_1 ... C_5, p^2$), like our general solution (4). However, it is easy to verify that a redefinition of the constants can easily make that solution independent of C_2. The constancy of the azimuthal angle $\phi = C_5$ indicates immediately that we are dealing with the special case $n_2/n_1 =$ const. I wish to thank M. V. Polyakov for calling my attention to Ref. 3.

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