Induced magnetic moment of neutrinos in magnetically ordered media

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The spatially odd part of the interaction of a Dirac neutrino with a transverse electromagnetic field in a magnetically ordered medium reduces to the magnetic form factor of the neutrino. In the static case, this form factor corresponds to an induced normal magnetic moment $\beta = 10^{-10}\beta_B$, where β_B is the Bohr magneton.

A neutrino acquires an induced charge and an induced magnetic moment in isotropic media with free charge carriers.[1-3] It is natural to expect that incorporating spin variables in a magnetically ordered system should lead to an induced magnetic moment of the neutrino significantly larger than that in Ref. 3, since in a magnetically ordered medium there is a net magnetic-moment density even in the ground state. This net density is produced by the oriented spins of the electrons.

In the present letter we show that in a situation in which the magnetic permeability of the medium is significantly different from unity the contribution of the axial (parity-breaking) current of electrons to the effective electromagnetic vertex of the neutrino reduces to the magnetic form factor. In the quasistatic case ($\omega \to 0$, $\mathbf{k} \to 0$) this form factor corresponds to an induced normal magnetic moment of the neutrino which is several orders of magnitude greater than the induced magnetic moment found in Ref. 3.

Examining the motion of massless soft neutrinos ($q^2 \ll M_w^2$) in a magnetically ordered medium, we can use the point four-fermion interaction of neutrinos with the electrons of the medium[4]:

$$\hat{H}_{\nu e} = -\sqrt{2}G_F\left(\bar{\psi}_\nu \frac{1+\gamma_5}{2}\gamma_\mu \psi_\nu\right)\left(\bar{\psi}_e \frac{1+4\xi+\gamma_5}{2}\gamma^\mu \psi_e\right). \quad (1)$$

We have written this interaction as a product of weak neutrino and electron currents. The parameter $G_F = 10^{-5}/m_p^2$ (m_p is the mass of the proton) and the parameter $\xi = \sin^2\theta_W$ (θ_W is the Weinberg angle) in expression (1) incorporate the interaction through charged and neutral currents. Here the operators ψ_e and ψ_ν represent the free electron and neutrino fields in the Schrödinger picture. The operator representing the weak current of electrons in (1) contains two terms, one proportional to the vector current $j_\mu^v = (\bar{\psi}_e \gamma_\mu \psi_e)$, and the other to the pseudovector current $j_\mu^a = (\bar{\psi}_e \gamma_5 \gamma_\mu \psi_e)$ of the electrons. Taking an average of the electron current over a volume element of the medium which is physically infinitely small, we find[5]

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$$j_\mu^\nu(\mathbf{r},t) = \int \frac{d^3p}{E}\, p_\mu \operatorname{Tr} \hat{f}(\mathbf{p},\mathbf{r},t) \tag{2}$$

$$\mathbf{j}^a(\mathbf{r},t) = \int d^3p \operatorname{Tr}(\vec{\Sigma}\hat{f}(\mathbf{p},\mathbf{r},t)) \quad , \tag{3}$$

where[1)] $p^\mu = (E, \mathbf{p})$, $E = \sqrt{\mathbf{p}^2 + m_e^2}$, $f_{\alpha\beta}(\mathbf{p}, \mathbf{r}, t)$ is the locally equilibrium one-particle density matrix of the electrons of the medium, $\vec{\Sigma} = \gamma_0\gamma_5\vec{\gamma}$ is the three-dimensional spin operator of the electron, and $\mathbf{S}(\mathbf{p},\mathbf{r},t) = \mathrm{Sp}(\vec{\Sigma}\hat{f})$ is the phase density of the spin in the medium.

As we know, the vector current of electrons (the conductivity current) is related to the electromagnetic field potentials A^μ through the polarization operator $\Pi_{\rho\mu}$. The axial current of electrons, (3), is by definition twice the macroscopic spin density in the medium, which can be associated with the magnetization. Using (3), we can thus write interaction (1), averaged over a volume of the medium which physically is infinitely small, as the effective operator for the electromagnetic interaction of neutrinos:

$$\hat{H}_{\nu A} = -e(\bar{\psi}_\nu \Gamma_\mu \psi_\nu)A^\mu \ , \tag{4}$$

where the neutrino electromagnetic vertex Γ_μ is

$$\Gamma_\mu(\omega,\mathbf{k}) = \frac{G_F(1+\gamma_5)}{8\pi\sqrt{2}\alpha}\gamma^\rho \left\{(1+4\xi)\Pi_{\rho\mu} + 2im_e\,\delta_{\mu i}\delta_{\rho j}(\delta_{il} - \mu_{il}^{-1})k_n e_{iln}\right\}. \tag{5}$$

$\mu_{il}(\omega, \mathbf{k})$ is the magnetic permeability tensor of the medium, and m_e is the mass of an electron. The first term in (5) has been discussed in several places.[1-3]

In contrast with those other papers, we are interested here in the more general case in which there is a dispersion in not only the dielectric constant $\epsilon_{ij}(\omega, \mathbf{k})$ but also the magnetic permeability of the medium, $\mu_{ij}(\omega, \mathbf{k})$. In this connection, the polarization operator contains an additional term, which is related to the spin current in the medium. This term is proportional to curl$M$. The contribution of the second term in (5) to the effective current of the transition has only a vector component:

$$eJ_{12}(\omega,\mathbf{k}) = \frac{eG_F m_e}{2\pi\sqrt{2}\alpha}\left[i\mathbf{k}\,\frac{(\bar{\nu}_L'\,\mathbf{g}\nu_L)}{2\sqrt{\epsilon_1\epsilon_2}}\right], \tag{6}$$

where the matrix vector $g_i(\omega, k) = \gamma_j\ (\delta_{ij} - \mu_{ij}^{-1})$ is taken in brackets of neutrino bispinors, $\nu_L = (1-\gamma_5)/2\nu$, and $\epsilon = |\mathbf{p}|$ is the energy of the neutrino. Expression (6) describes the contribution to the transition current from the induced magnetic form factor of the neutrino:

$$\beta_i^\nu(\omega,\mathbf{k}) = \frac{eG_F m_e}{2\pi\sqrt{2}\alpha}\,\frac{(\bar{\nu}_L'\gamma_j\nu_L)}{2\sqrt{\epsilon_1\epsilon_2}}\,(\delta_{ij} - \mu_{ij}^{-1}(\omega,\mathbf{k})). \tag{7}$$

It follows from (7) that in the quasistatic limit the induced normal magnetic moment of the neutrino is $(k=\mathbf{p}_1-\mathbf{p}_2\to 0,\ \omega=\epsilon_1-\epsilon_2\to 0)$

$$\beta_i^\nu = \frac{eG_F m_e}{2\pi\sqrt{2\alpha}}\, n_j\,(\delta_{ij} - \mu_{ij}^{-1}(0,0)) \ . \tag{8}$$

For a polycrystalline ferromagnetic material, in an external magnetic field considerably weaker than the saturation field, we can set $\mu_{ij}=\mu\delta_{ij}$, where $\mu \gg 1$. In this case the induced magnetic moment

$$\vec{\beta}^\nu = \frac{eG_F m_e}{2\pi\sqrt{2\alpha}}\,\mathbf{n} \qquad (\mathbf{n} = \mathbf{p}/|\mathbf{p}|\ ) \tag{9}$$

is directed along the momentum of the neutrino and has a magnitude

$$\beta^\nu = 10^{-5}\,\frac{m_e^2}{\sqrt{2\pi\alpha}\,m_p^2}\,\beta_B \approx 10^{-10}\,\beta_B\ . \tag{9'}$$

If the ferromagnet is in a strong magnetic field B $(B \gg 4\pi M_0)$, the neutrino energy

$$-\vec{\beta}^\nu\mathbf{B} = -\frac{em_e G_F}{2\pi\sqrt{2\alpha}}\,\mathbf{n}(\mathbf{B}-\mathbf{H}) = -4\pi\frac{em_e G_F}{2\pi\sqrt{2\alpha}}\,\mathbf{n}\mathbf{M} \tag{10}$$

does not depend on the strength of the magnetic field, since the magnetization of the medium, **M**, reaches saturation. In this case we obviously have **B**||**M**, so the induced magnetic moment

$$\beta^\nu = 4\pi\frac{em_e G_F}{2\pi\sqrt{2\alpha}}\,\frac{M}{B}\,\mathbf{n} \tag{11}$$

is inversely proportional to the magnetic field B.

In a ferromagnetic crystal, in which the magnetic permeability μ_{ij} is anisotropic, the induced magnetic moment will no longer be directed along the momentum of the neutrino. The direction of the magnetic moment also depends on the magnitude and direction of the magnetic field with respect to the anisotropy axis of the crystal. In this case, however, the magnetic moment is again given in order of magnitude by (9′).

[1])We are using a system of units with $\hbar=c=1$, the Feynman metric $q_\mu q^\mu=\omega^2-\mathbf{k}^2$, and the standard representation for the Dirac γ matrices; here $\gamma_5=\gamma_5^+ = i\gamma^0\gamma^1\gamma^2\gamma^3$, $e^2=\alpha=1/137$.

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