Alfvén shock wave trains with a dispersion

C. F. Kennel,[1)] M. A. Mal'kov, R. Z. Sagdeev, V. D. Shapiro, and A. V. Khrabrov

Institute of Space Research, Academy of Sciences of the USSR

(Submitted 11 May 1988)

Pis'ma Zh. Eksp. Teor. Fiz. **48**, No. 2, 75–78 (25 July 1988)

Solutions in the form of nonlinear structures—shock waves—are obtained for Alfvén waves which are excited by a long-wavelength source and which propagate along the magnetic field. The dispersion and viscosity are taken into account in the solution.

Intense MHD fluctuations observed in solar wind in front of the leading edge of a shock wave are caused by the cyclotron instability of an energetic-particle beam.[1,2] In the case of a comet shock wave the heavy-ion beam is formed as a result of photoionization of a gas flow that originates in the nucleus of the comet.[3,4] The MHD oscillations in front of the leading edge of a shock wave of a planetary magnetosphere are excited by the energetic protons of the solar wind which are reflected from the shock front.[5] Finally, similar phenomena occur in front of the leading edge of a shock wave of cosmic rays.[6] The MHD fluctuations in this case are excited by the accelerated particles which propagate through the shock front toward interstellar plasma.

In all the cases mentioned above the cyclotron instability leads to an excitation of long-wavelength Alfvén waves with the wave numbers $k_0 \sim \omega_{Hi}/v$ (ω_{Hi} is the cyclotron frequency of the energetic particles, v is the velocity of these particles, and $v \gg v_A$, where v_A is the Alfvén velocity). A nonlinear torsion of the wave spectrum gives rise to short wavelength scales $k \sim \omega_{Hp}/v_A$ (ω_{Hp} is the cyclotron frequency of the solar wind protons) and to a state in which the absorption of Alfvén waves by thermal protons becomes important. This situation leads to the formation of a characteristic wave structure which can be called Alfvén shock wave trains. In the long-wavelength part of such a wave train the wave is usually polarized linearly and sometimes elliptically. As a result of torsion, a short-wavelength oscillatory structure forms in the front part of the wave profile. This structure is always a counterclockwise-polarized circular wave.

In the present letter we will show that a structure of this sort can be obtained for Alfvén waves which propagate along the external magnetic field if the nonlinear effects of density modulation by the rf pressure, as well as the dispersion and viscosity, are taken into account. The viscosity simulates the damping of short-wavelength Alfvén waves.

Taking into account the effects mentioned above, we can write the following equation for the magnetic field components of the Alfvén wave[7]:

$$\frac{\partial b}{\partial \tau} + \frac{\partial}{\partial \zeta}(b|b|^2) + \epsilon(i-\nu)\frac{\partial^2 b}{\partial \zeta^2} = \frac{1}{2\pi}\int \Gamma(\zeta-\zeta')b(\zeta')d\zeta'. \qquad (1)$$

In this equation we used the following dimensionless variables:

$$b = \frac{B_x + iB_y}{B_0}, \qquad \tau = \frac{1}{4}\frac{\omega_{Hi} t v_A}{v}, \qquad \zeta = \frac{\omega_{Hi}(z - v_A t)}{v}, \tag{2}$$

$$\epsilon = 2\frac{v_A}{v}\frac{\omega_{Hi}}{\omega_{Hp}}, \qquad \nu = 2R_0\frac{\omega_{Hp}}{v_A},$$

where B_0 is the external magnetic field, and R_0 is the effective mean free path. The small parameter ϵ gives the ratio of the characteristic dispersion (dissipative) length scale to the instability scale. In writing the source on the right side of (1) we have assumed that because of the cyclotron instability, only the linearly polarized first harmonic of the waves is excited. Accordingly, we assume

$$\Gamma(\zeta) = Se^{i(\zeta+\varphi)} + S^* e^{-i(\zeta+\varphi)}, \tag{3}$$

where $Se^{i\varphi} = \gamma_1 - i\Delta\omega_1$, and γ_1 and $\Delta\omega_1$ are the increment and the shift of the pump frequency, expressed in dimensionless time units determined from (2).

We seek steady-state solutions of Eq. (1) in the form

$$b(\zeta, \tau) = a(\xi) e^{i\theta(\xi)}, \qquad \xi = \zeta - u\tau .$$

From (1) we then find the following system of equations for the amplitude and phase of the wave:

$$a(a^2 - u) - \epsilon a\frac{d\theta}{d\xi} - \nu\epsilon\frac{da}{d\xi} = 2Sa_1 \sin\chi\cos\theta,$$
$$\epsilon\frac{da}{d\xi} - \nu\epsilon a\frac{d\theta}{d\xi} = -2Sa_1\sin\chi\sin\theta, \qquad \chi = \xi + \alpha + \varphi . \tag{4}$$

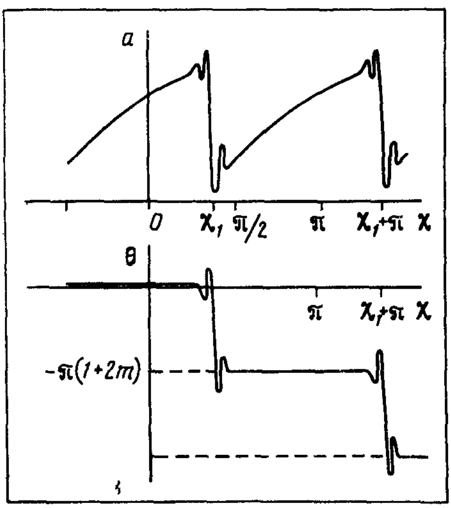

FIG. 1.

In these equations a_1 and α are the amplitude and phase of the unstable harmonic:

$$a_1 e^{i\alpha} = \frac{1}{2\pi} \int_{-\pi}^{\pi} b(\xi) e^{-i\xi} d\xi . \tag{5}$$

The solution of the basic equation (1) for this case is shown in Fig. 1. This equation consists of linearly polarized parts, in which the amplitude varies smoothly and which are divided by discontinuities (shock wave fronts). At $\chi = \chi_1$ and $\chi = \pi + \chi_1$ the position of the discontinuities is chosen so that the condition $\langle b \rangle = (1/2\pi)\int_{-\pi}^{\pi} b(\xi) d\xi = 0$ would be satisfied. In the regions in which the solution varies smoothly the amplitude and phase are determined from the relations

$$a = 2\sqrt{\frac{u}{3}} \sin\frac{\chi + \pi}{3}, \qquad \theta = 0, \qquad 0 < \chi < \chi_1$$

$$a = 2\sqrt{\frac{u}{3}} \sin\frac{\chi}{3}, \qquad \theta = -\pi, \qquad \chi_1 < \chi < \pi + \chi_1$$

$$a = 2\sqrt{\frac{u}{3}} \sin\frac{\chi - \pi}{3}, \qquad \theta = 0, \qquad \chi_1 + \pi < \chi < 2\pi .$$

Solutions of this type can be found if the following condition for the nonlinear phase-velocity shift is satisfied:

$$u = 3(Sa_1)^{2/3}. \tag{6}$$

Within the limits of the discontinuity ($\chi \approx \chi_1$) the evolution of the magnetic field can be described by a system of Hamilton's form of equations of motion, with allowance

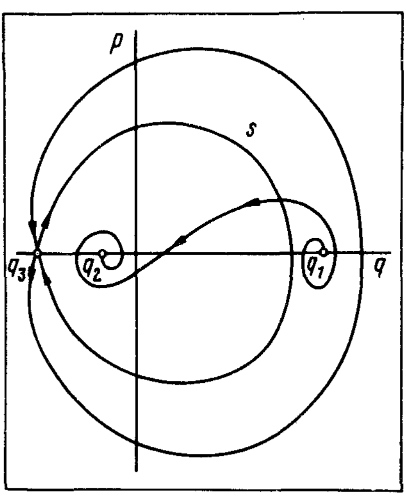

FIG. 2.

for dissipation

$$\frac{dq}{d\eta} = \frac{\partial H}{\partial p} + \nu\frac{dp}{d\eta} \qquad \frac{dp}{d\eta} = -\frac{\partial H}{\partial q} - \nu\frac{dq}{d\eta}, \qquad \eta = \frac{\chi - \chi_1}{\epsilon}, \tag{7}$$

where $q = a\cos\theta$, $p \equiv a\sin\theta$, and the role of the Hamiltonian is performed by the function

$$H = \frac{u}{2}(p^2 + q^2) - \frac{1}{4}(p^2 + q^2)^2 + 2\left(\frac{u}{3}\right)^{3/2} q\sin\chi_1 .$$

The phase diagram of system (7) at $\nu = 0$ and the trajectory for $\nu > 0$ are shown in Fig. 2. This system of equations has two fixed elliptic points $q = q_1, q_2; p = 0$ and one hyperbolic point $q = q_3$; $p = 0$, $q_n = 2\sqrt{u/3}\sin[\chi_1 + (2\pi n - \pi)/3]$. As a result of dissipation, the system transforms from an unstable equilibrium $a_{\max} = q_1$ to a stable equilibrium $a_{\min} = -q_2$. The total change in the phase within the discontinuity is $-\pi(1 + 2m)$, where m is an integer. The direction of rotation near the stable equilibrium $a_{\min}$ is opposite to that near the unstable equilibrium $a_{\max}$. After the system crosses the separatrix S (Fig. 2), the phase monotonically. This process corresponds to the counterclockwise polarized wave within the shock front, in which the magnetic field rotates clockwise, while the wave profile, along with the plasma, moves with respect to the observer.

Using relation (5), we can relate the pump amplitude and phase a_1 and φ to the position of the discontinuity χ_1. In particular, the solution with a zero frequency shift ($\varphi = 0$) would be of interest:

$$\chi_1 = \frac{\pi}{4} + \frac{3}{2}\arcsin\frac{\sqrt{3}-1}{2}, \qquad u = 3.154\,\gamma, \quad a_{max} = 1.889\sqrt{\gamma}.$$

We wish to thank V. I. Shevchenko and V. L. Galinskiĭ for furnishing the results of the numerical modeling of Eq. (1). These results are largely similar to those described in this letter.

[1)]University of California, Los Angeles.

[1]C. S. Wu and R. C. Davidson, Geophys. Res. **77**, 5399 (1972).
[2]R. Z. Sagdeev, V. D. Shapiro, V. I. Shevchenko, and K. Szego, Geophys. Res. Lett. **13**, 85 (1986).
[3]B. T. Tsurutani and E. J. Smith, Geophys. Res. Lett. **13**, 263 (1986).
[4]K. Numoto *et al.*, in: Proceedings of Heidelberg Symp. on the Exploration of Halley's Comet, ESA SP-250, 1986, p. 249.
[5]T. Hada, C. F. Kennel, and T. Terasawa, J. Geophys. Res. **91**, 4471 (1986).
[6]W. I. Axford, Proceedings of the Seventeenth ICRC **12**, 155 (1981).
[7]A. Rogister, Phys. Fluids **14**, 2733 (1971).

Translated by S. J. Amoretty