Dynamic stochastization of electrons in semiconductors with superlattices

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Stochastization of current carriers in semiconductor superlattices in an external electromagnetic field has been detected. The characteristics of the external fields and the parameters of the semiconductors, which are necessary to achieve this effect, are estimated. The macroscopic manifestations of this effect are discussed.

Semiconductors with superlattices are one of the most interesting and promising subjects of present-day solid-state physics. Despite the fact that these semiconductors have been studied extensively (see, for example, the review article by Bass and Tetervov[1]), the enduring interest in superlattices is attributable to the wealth of physical properties of this system which are fundamentally not obtainable in conventional semiconductors.

Assuming, for example, that the charge carriers with an effective mass m^* are situated only in the lower minizone of width Δ, we can use the following expression to accurately approximate the dispersion relation of these carriers[1]:

$$\epsilon = \epsilon_0 + (p_x^2 + p_y^2)/2m^* - \Delta\cos(p_z d/\hbar),$$

where ϵ_0 is the position of the minizone, d is the period of the superlattice, and the z axis runs in the direction of the superlattice axis.

In the present letter we will demonstrate that a new effect, which involves the stochastization of electrons in an external field, can in principle be observed.

The equation for cyclotron oscillations in an external electromagnetic field, whose constant component h_0 is directed perpendicular to the superlattice axis, is the equation for the mathematical pendulum under the influence of an external force.[1] This equation can be written in the form

$$\dot{H} = f(\kappa(H;\Theta);\, t), \qquad \dot{\Theta} = \omega(H), \tag{2}$$

where $H = \frac{1}{2}\dot{\kappa}^2 - W_0^2\cos\kappa$ is the adiabatic integral with $f = 0$, Θ is the oscillation phase,

$$\kappa = P_z d/\hbar = 2\arcsin((H/2W_0^2)^{1/2}\sin(\Theta;(H/2W_0^2)^{1/2})),$$

$$W_0 = edh_0\sqrt{\Delta}/\cos\hbar\sqrt{m^*}, \qquad W(H) = \pi W_0/2K(\sqrt{H/2W_0^2}).$$

Here $K(\alpha)$ is the elliptic integral and $dH/dt \equiv \dot{H}$.

The behavior of $f(\kappa;t)$, a perturbation in a weak alternating field, is generally determined by the characteristics and geometry of this field and by the parameters of the superlattice. To illustrate this effect, we restrict the discussion to just one type of perturbation.

Let us consider an electromagnetic wave propagating along the axis of the superlattice

$$\mathbf{E} = (0;\ E_y\ ;-\ \frac{\epsilon_{yz}}{\epsilon_{zz}} E_y)\cos(\omega t - kz), \quad \mathbf{H} = (h_0 - \frac{kc}{\omega} E_y\ ;\ 0;\ 0)\cos(\omega t - kz),$$

where ϵ_{ij} is the dielectric tensor. The external force $f(\kappa;t)$ in this case is

$$f(\kappa;t) = -\frac{h_0 E_y e^2 d}{m^* c\hbar}\ \omega(H)\cos(\omega t - kz), \tag{3}$$

and the conditions under which this force is small are $E_y \ll h_0\hbar/m^*cd$, $\omega \ll \omega_0$.

The dynamic system (2) near the separatrix ($H = 2W_0^2$) is known to behave chaotically[2] in a certain energy region of the external parameters. To find this energy region, we will make use of the Chirikov criterion.[2,3] In the case of the perturbation chosen, (3), this criterion is

$$0 \leqslant \delta\,|\ln\delta\,| \leqslant 3\pi\frac{eE_0}{\omega\sqrt{m^*\Delta}}\ , \tag{4}$$

where $\delta = 1 - H/2W_0^2$.

Stochastization accounts for a rapid mixing (at a rate on the order of $\tau_\Theta \approx W_0^{-1}$) of the phases[3] Θ. The distribution function $P(H;t)$ is introduced to describe the dynamics of the system at times $t \gg \tau_\Theta$, which involves a slow destruction of the integrals of motion H. In the random-phase approximation this function satisfies the Fokker–Planck–Kolmogorov equation[3] with the diffusion coefficient

$$D(H) = \pi\frac{de^3 h_0 E_y^2}{\hbar c m^{*3/2}\Delta^{1/2}}\sqrt{2H}\,/\,K(\sqrt{H/2W_0^2}\,). \tag{5}$$

The Fokker–Planck–Kolmogorov equation has a new time scale $\tau_H \approx 10^9\tau_\Theta \times (W_0/W)^4(E_y/h_0)^2$. This time scale corresponds, as will be seen below, to the energy redistribution of electrons.

Since stochasticity is a qualitatively different kind of motion of dynamic nonlinear systems, the kinetic properties of the superlattices should also change qualitatively. The equation of motion (2) describes the characteristic features of the kinetic equation for the electron distribution function $f(\mathbf{rp}t)$ in the relaxation-time approximation. Consequently, if condition (4) is satisfied, the characteristic features become randomized. This randomization in turn leads to a momentum and energy redistribution of electrons.

Randomization of phases of the nonlinear oscillator (2) corresponds to a uniform momentum distribution with respect to the direction. The stochasticized carriers therefore have no effect on the transport processes. The y and z components of the conductivity tensor of the superlattices, for example, are expected to decrease, since the stochastization in the geometry suggested here involves the y and z projections of the quasimomentum.

In calculating the kinetic coefficients we now integrate over the energy only to the energy ϵ_S corresponding to the lower boundary to the stochastic layer, rather than over the entire minizone. To estimate the current under these conditions, we can use the equation

$$I_S = I_R \Gamma, \Gamma = \int_0^{\epsilon_S} \epsilon^{3/2} f(\epsilon) d\epsilon \Big/ \int_0^{\Delta} \epsilon^{3/2} f(\epsilon) d\epsilon \ , \qquad (6)$$

where I_R is the current in which the stochastization is ignored and $f(\epsilon)$ is the Fermi distribution function.

Correct mathematical determination of ϵ_S is a very complex problem which does not have a fundamental solution at this time.[2,3] We will therefore consider its estimated value, assuming that the region of global stochasticity is directly contiguous to the region of regular motion (the region of local stochasticity is ignored in this case).[3]

In the quasimomentum space the ceiling of the minizone Δ is equal to the separatrix of Eq. (2). The lower boundary of the stochastic layer ϵ_S is determined by the width of the overlapping resonances δ. Assuming the width of the layer to be small to the extent that the perturbation is small and expanding the dispersion relation in a series, we obtain $\epsilon_S = \Delta(1 - \delta^2)$.

The next step of the kinetics of the process involves finding an equal energy distribution of the electrons stochasticized as a result of the destruction of the integrals of motion. The time scale of this process is τ_H.

Here are some estimates. For the superlattices with $d \approx 100$ Å, $\Delta = 50$ meV, $m \approx 0.1 m^*$ and fields $h_0 \approx 100$ G, $E \approx 1$ V/cm, $\omega \approx 10^9$ Hz, and $T \approx 300$ K, we have $\tau_\Theta \approx 10^{-11}$ s (for comparison, we note that τ_Θ is on the order of the relaxation time), $\tau_H \approx 10^{-2}$ s, $\delta \approx 0.2$, $\epsilon_S \approx 48$ meV, and $\Gamma \approx 0.89$.

In describing the stochastization of the carriers in the superlattices we ignored the effect of the change in the conductivity on the propagation of the perturbing electromagnetic wave. This effect, appreciable in the case of strong fields, requires a self-consistent analysis. In general, electrons from other minizones and deviations from random-phase approximation caused by the boundaries of the stochastic layer and by the islands of stability should also be taken into account. Principally numerical calculations are now being used for this purpose. These factors and also other types of perturbations which give rise to stochastization of the carriers in the superlattices will be analyzed in a separate paper.

[1]F. G. Bass and A. P. Tetervov, Physics Reports **140**, 239 (1986).
[2]G. M. Zaslavskiĭ, *Stochastic Behavior of Dynamic Systems*, Nauka, Moscow, 1984, p. 271.
[3]A. Likhtenberg and M. Liberman, *Regular and Stochastic Dynamics*, Mir, Moscow, 1984, p. 528.

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