Universal instability of two-dimensional vortices in a plasma

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The switch to a kinetic description of the plasma results in a growth of negative-energy waves at the boundary of a vortex.

The possible role of vortices in the mixing of a plasma has stimulated a stream of papers on vortex theory. Many types of vortices have turned out to be stable in the hydrodynamic description, i.e., if the velocity spread of the particles is ignored (see the reviews[1–3]). Most stable vortices realize a maximum of the energy at fixed values of the other integrals of motion, and it has accordingly been suggested that incorporating dissipation may result in a growth of vortices.[1] However, incorporating dissipation results primarily in an excitation of oscillations of the boundary of a vortex which have a negative energy (with respect to the energy of an unperturbed vortex), and if there is a dissipation they grow, rather than being damped. In addition to collisional dissipation, an instability may result from a resonant absorption of energy by hot particles, i.e., Landau damping. A finite Larmor radius was taken into account in Ref. 4, but in an approximation in which this factor does not yet introduce any new effects.

In the present letter we wish to examine an instability caused by Landau damping. We assume that the following conditions hold: a) There are few hot particles, and they do not affect the polarization or frequency of the natural oscillation modes of the boundary of a vortex. b) The nonuniformity of the plasma in its unperturbed state can be ignored in the calculation of the resonant energy absorption by hot particles.

The procedure for calculating the instability of a natural mode is as follows. We first find the frequency ω, the energy W, and the electric field of the natural mode in the hydrodynamic approximation. We then calculate the rate of resonant energy absorption by hot particles in the field which has been found, $\dot{W}$. The instability growth rate γ is

$$\gamma = -\dot{W}/W . \tag{1}$$

We will demonstrate the feasibility of this procedure by calculating the instability of a two-dimensional electron vortex in which the vorticity [the curl of the generalized angular momentum of the electrons, $\mathrm{curl}\mathbf{P} = \mathrm{curl} m\mathbf{v}_e) + e\mathbf{H}/c$] is $\mathbf{e}_z(\Omega + \delta\Omega)$ inside a vortex and $e_z(\Omega - \delta\Omega)$ outside it.[3]

The equations of ideal electron MD are used as the equations for the cold particles.[3] In the two-dimensional case in which we are interested here, the vorticity is $\Omega = e_z\Omega(x,y)$, and the equation for Ω takes the form of an equation describing a freezing of Ω in the electrons:

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$$\partial\Omega/\partial t + (\mathbf{v}_e, \nabla)\Omega = 0, \qquad \Omega = \frac{e}{c} H - \lambda^2 \frac{e}{c} \Delta H, \tag{2}$$

where $\lambda^2 = c^2/\omega^2_{pe}$.

We restrict the discussion to the case of perturbations whose wavelength is short in comparison with the size of the vortex. We direct the y axis along the boundary of the vortex, and the x axis along the perpendicular to this boundary. We seek natural oscillation modes in the form of sinusoidal displacements of the vortex boundary. We then have $\Omega(r,t) = \Omega_0(r) + 2\Gamma_k \Delta\Omega \sin(\kappa y - \omega t)$, where κ is the wave number, and Γ_κ is the amplitude of the excursion of the vortex boundary from its equilibrium position ($\Gamma_\kappa \kappa \ll 1$). Substituting $\Omega(\mathbf{r}, t)$ into (2), we find

$$\omega = (\delta\Omega/m)\kappa\lambda\left(\frac{1}{\sqrt{1+\kappa^2\lambda^2}} - 1\right). \tag{3}$$

Using the equations of electron MHD, we find the electric field of the natural mode. For simplicity we assume $\delta\Omega \ll \Omega$. The vortex part of the field is then much weaker than the potential part, and the electric potential is

$$\varphi(\mathbf{r}, t) = \Gamma_\kappa \lambda \frac{\delta\Omega\,\Omega \exp(-|x|\sqrt{\kappa^2 + \lambda^{-2}})}{m\, e\sqrt{1+\kappa^2\lambda^2}} e^{i\kappa y - i\omega t}. \tag{4}$$

It is not difficult to show that the energy of the electric field of the oscillations is small in comparison with the magnetic field energy and the kinetic energy of the electrons. The sum of the latter can be expressed in terms of the frozen-in quantity $\Omega(\mathbf{r}, t)$:

$$W = c^2/(8\pi e^2) \int \Omega(\mathbf{r},t)\,\Omega(\mathbf{r}', t) K_0(|\mathbf{r}-\mathbf{r}'|/\lambda) d^2\mathbf{r}' d^3\mathbf{r}, \tag{5}$$

where K_0 is a modified Bessel function. Substituting in $\Omega(\mathbf{r}, t)$, we find the energy of the natural oscillation mode of a plane vortex:

$$W = \frac{1}{8} S\Gamma^2_\kappa n\lambda \frac{(\delta\Omega)^2}{m}\left(\frac{1}{\sqrt{1+\kappa^2\lambda^2}} - 1\right). \tag{6}$$

Here S is the surface area of the boundary between regions with different vorticities.

Using the potential which we have calculated, let us now find the power absorbed by the resonant particles. The effect of the magnetic field on the hot particles can be ignored under the condition $\mathbf{r}_H \gg 1$. If, in accordance with condition b), we ignore the nonuniformity of the hot-particle distribution in the unperturbed vortex, then we can write (Ref. 5, for example)

$$\dot{W} = -\frac{2}{(2\pi)^4} \omega \int k^2 |\varphi(\mathbf{k})|^2 \epsilon''(\omega, \mathbf{k}) d^3\mathbf{k}, \tag{7}$$

where $\varphi(\mathbf{k}) = \int \varphi(\mathbf{r},t) e^{-i\mathbf{k}\mathbf{r}} d^3\mathbf{r}$, and $\epsilon''(\omega,\mathbf{k})$ is the imaginary part of the dielectric constant of the plasma. If the hot particles are isotropic (we are thinking of particles which are responsible for a resonant absorption of energy; they could be electrons or ions), we find, using (1), (5), and (7),

$$\gamma = \int F'(v)\gamma(v)dv, \tag{8}$$

$$\gamma(v) = \frac{1}{4\pi^2}\frac{\Omega^2}{m^2}\frac{\omega_p^2}{\omega_{pe}^2}\frac{\sqrt{1+\kappa^2\lambda^2}\,(\sqrt{1+\kappa^2\lambda^2}-1)^{-1}}{(1+v^2/(\lambda^2\omega^2))^2\sqrt{\kappa^2\lambda^2+\omega^2\lambda^2/v^2}}, \tag{9}$$

where ω is the plasma frequency of the hot particles, and $F(v)$ is their distribution function projected onto an arbitrary axis.

In using the expression "universal instability" we have attempted to emphasize the similarity between the instability found here and a drift instability.[6] In each case, the energy source is a nonuniformity (in our case, caused by the vortex) which leads to the existence of negative-energy waves. The dissipation required for the growth can almost always be found.

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