Fluctuations of the diamagnetic susceptibility of disordered metals

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The fluctuations which arise in the diamagnetic susceptibility of disordered metals because of the random distribution of impurities in them have been studied. These fluctuations are shown to be substantially larger than the susceptibility averaged over realizations of a random potential.

The expression for the thermodynamic potential of the conduction electrons in a static magnetic field is ordinarily used to calculate the diamagnetic susceptibility of metals. We believe, however, that a different method would be more convenient for dealing with the effect of collisions of conduction electrons with impurity centers on the Landau diamagnetism and for studying fluctuations in the susceptibility which stem from the random distribution of impurities. Specifically, one could study the response of the system to a static external magnetic perturbation. If a field with a vector potential $\mathbf{A} = \mathbf{a}\exp(i\mathbf{kr})$ is applied to the system of conduction electrons in the metal, and if this field satisfies the gauge condition $(\mathbf{kA}) = 0$, the diamagnetic current is given, in first order in $\mathbf{A}$, by[1]

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$$\mathbf{j} = -i\frac{e^2}{m^2}\int\frac{d\mathbf{p}d\epsilon}{(2\pi)^4}(1-\tanh\frac{\epsilon}{2T})\{\mathbf{p}(\mathbf{p}\mathbf{A})[G_\epsilon^R(\mathbf{p}_+)G_\epsilon^R(\mathbf{p}_-) - G_\epsilon^A(\mathbf{p}_+)G_\epsilon^A(\mathbf{p}_-)]$$

$$+ m\mathbf{A}[G_\epsilon^R(\mathbf{p}) - G_\epsilon^A(\mathbf{p})]\}, \quad (1)$$

where $\mathbf{p}_\pm = \mathbf{p} \pm \mathbf{k}/2$. The retarded and advanced Green's functions of an electron, averaged over the random distribution of impurities, in (1) are given by

$$G_\epsilon^{R(A)}(\mathbf{p}) = (\epsilon - \mathbf{p}^2/2m + \mu \pm i/2\tau)^{-1}$$

where μ is the chemical potential, and τ is the momentum relaxation time of the electrons (here and below, we are using a system of units with $\hbar = c = k_B = 1$). It is easy to see that the expansion of (1) in powers of the vector $\mathbf{k}$ begins with the quadratic term. Carrying out the expansion, and restricting the analysis to the term quadratic in the wave vector, we find

$$\mathbf{j} = -\frac{ie^2}{4m^3}\int\frac{d\mathbf{p}d\epsilon}{(2\pi)^4}(1-\tanh\frac{\epsilon}{2T})\mathbf{p}(\mathbf{p}A)\{([G_\epsilon^R(\mathbf{p})]^{-3} - [G_\epsilon^A(\mathbf{p})]^3)\mathbf{k}^2$$

$$+ ([G_\epsilon^R(\mathbf{p})]^4 - [G_\epsilon^A(\mathbf{p})]^4)\frac{(\mathbf{p}\mathbf{k})^2}{m}\}. \quad (2)$$

It follows immediately from this expression that Landau's result for the diamagnetic susceptibility,

$$\mathbf{j} = \chi_L \mathbf{k}^2\mathbf{A} = -\frac{e^2 p_F}{12\pi^2 m}\mathbf{k}^2\mathbf{A}, \quad (3)$$

holds over broad ranges of the temperature and density of impurity centers. The only restriction on the density of the impurities is the condition $\epsilon_F\tau \gg 1$ (ϵ_F is the Fermi energy, and p_F is the Fermi momentum). The reader is referred to Ref. 2 for further information regarding this assertion.

It has recently been learned that the conductivity of a specific sample with a given random distribution of impurities may differ substantially from the conductivity averaged over realizations of a random potential.[3-5] This assertion also applies to certain other characteristics of a disordered metal.[6-8]

The fluctuations in the diamagnetic susceptibility are particularly large in disordered conductors. To calculate the fluctuations in the diamagnetic susceptibility, we should average the square of current density (2) over the impurities. It is not difficult to show that the average expression for the square of the current is dominated by the terms which contain Green's functions raised to the fourth power. After going through the procedure of taking an average and integrating over the momenta of the electrons, we find the following expression for the average square current in the limit of a dirty metal, $T\tau \ll 1$:

$$\langle \mathbf{j}^2 \rangle = \frac{16e^4 v_F^8}{27} (\pi \nu \tau^2)^5 \tau \iint \frac{d\epsilon\, d\epsilon'}{(2\pi)^2} (1 - \mathrm{th} \frac{\epsilon}{2T})(1 - \tanh \frac{\epsilon'}{2T})$$

$$\times \mathrm{Re} \int (dq) [C^4_{\epsilon - \epsilon'}(\mathbf{q}) + D^4_{\epsilon - \epsilon'}(\mathbf{q})] \mathbf{k}^4 \mathbf{A}^2 \quad , \tag{4}$$

where v_F is the Fermi velocity, and $\nu = mp_F/2\pi^2$ is the state density of electrons near the Fermi energy. In the absence of a magnetic field and in the absence of magnetic impurities, a cooperon $C_\omega(\mathbf{q})$ and a diffusion $D_\omega(\mathbf{q})$ are equal:

$$C_\omega(\mathbf{q}) = D_\omega(\mathbf{q}) = \frac{1}{2\pi \nu \tau^2} (D \mathbf{q}^2 - i\omega)^{-1} \quad , \tag{5}$$

where $D = v_F^2 \tau/3 = l^2/3\tau$ is the diffusion coefficient, and l is the electron mean free path.

In (4), we have $(dq) = d^d q/(2\pi)^d L^{3-d}$, where d is the dimensionality of the sample. If the sample is a thin film of thickness $L \ll \sqrt{D/T} \sim 1/\sqrt{T\tau}$, we have $d = 2$. In the case of a thin filament, with a radius less than $l/\sqrt{T\tau}$, we would have $d = 1$.

After an integration over the energies ϵ and ϵ' in (4), we find

$$\langle \mathbf{j}^2 \rangle = \langle \chi^2 \rangle \mathbf{k}^4 \mathbf{A}^2 = \frac{4p_F^2 v_F^5 \tau^3}{3\pi T^2}$$

$$\times \chi_L^2 \int (dq) \sum_{n, n_1 = 0}^{\infty} (n + n_1 + 1 + D\mathbf{q}^2/2\pi T)^{-4} \mathbf{k}^4 \mathbf{A}^2 \tag{6}$$

Going through the integration and the summation in (6), we find the following expression for the average square susceptibility:

$$\langle \chi^2 \rangle = C_d \zeta(3 - d/2)(p_F l)^2 (l/L)^{3-d} (T\tau)^{d/2 - 2} \chi_L^2 \quad , \tag{7}$$

where $\zeta(x)$ is the Riemann zeta function. The numerical coefficients in (7) are $C_3 = (1/4)\sqrt{3/2\pi} C_2 = 2/3\pi, C_1 = 5/4\sqrt{6\pi}$. In the case of small particles, with a linear dimension smaller than $l/\sqrt{T\tau}$, we would have $C_0 = 4/3\pi$.

In the case of a low dimensionality, $d < 3$, it is assumed that the mean free path l is far shorter than all the linear dimensions of the sample. Otherwise, we should incorporate the collisions of electrons with the boundary of the sample. Accordingly, the condition $l \ll L \ll l\sqrt{T\tau}$ should hold for samples of low dimensionality. It follows from the final result, (7), that even in the case of a three-dimensional disordered metal the fluctuations in the susceptibility from sample to sample would be so large that the relation $\langle \chi^2 \rangle \gg \langle \chi \rangle^2 = \chi_L^2$ would hold.

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