Reflectionless quantum transport and fundamental ballistic-resistance steps in microscopic constrictions

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A quantization of the resistance of microscopic constrictions in a two-dimensional electron gas [B. J. van Wees *et al.*, Phys. Rev. Lett. **60**, 848 (1988); D. A. Wharam *et al.*, J. Phys. **21**, L209 (1988)] arises if the constriction has a sufficiently smooth shape: $\pi^2\sqrt{2R/d} > 1$, where d and R are the diameter and radius of curvature of the constriction. The shape of the steps is shown to be related to the geometry of the constriction.

Recent experiments on microscopic constrictions formed in two-dimensional electron layers have revealed an abrupt change in the conductance G as a function of the width of the constriction, d, adjusted by varying the voltage on a gate.[1,2] The sizes of the corresponding steps have turned out to be equal to the fundamental quantum $e^2/$

$\pi\hbar$. Wharam *et al.*[2] have related a similar pattern to a representation in terms of the electrical conductivity in a long channel under conditions of a transverse quantization of electrons. A one-dimensional picture of this sort, with well-defined transverse-quantization levels, does indeed lead to the appearance of steps of the required size in G. However, the independence of G from the length of the channel raises the question of how the accommodation regions at the end points, where this picture of the quantization of the transverse motion would be disrupted, contribute to the resistance. The effect of these regions would appear to be extremely dramatic, since estimates[3] put the aperture resistance in the case $d \sim k_F^{-1}$ at $\pi\hbar/e^2$ in order of magnitude. Furthermore, the geometry of the channel in Ref. 1 was definitely not approximately one-dimensional. There is accordingly the question of the particular conditions under which one can observe a clear picture of resistance quantization and its sensitivity to the geometry of the constriction.

In the present letter we show that the smoothness of the variation in the transverse dimension of the constriction plays a decisive role in determining whether the effect can be observed. The existence of universal steps in $G(k_F d)$ does not require a sharply bounded region with well-defined transverse-quantization levels.[2] The condition that the constriction be smooth, like the requirement on the length of the channel, turns out to be not very stringent, because of numerical factors.

A smooth variation $d(x)$ makes possible a reflectionless matching of electron states. The circumstance is seen in the possibility of an adiabatic separation of variables in the Schrödinger equation. If, in accordance with the experimental conditions of Refs. 1 and 2, we ignore the curvature of the bottom of the quantum well, we can write the wave function $\psi(x,y)$, which is the solution of the boundary-value problem

$$-\frac{\hbar^2}{2m}\Delta\psi = E\psi, \qquad \psi[y = \pm d(x)/2] = 0, \tag{1}$$

in the form $\psi = \psi(x)\varphi_x(y)$, where

$$\varphi_x(y) = [2/d(x)]^{1/2}\sin\{\pi n[2y + d(x)]/d(x)\} \tag{2}$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \epsilon_n(x)\psi = E\psi, \quad \epsilon_n(x) = \frac{\pi^2 n^2\hbar^2}{2md^2(x)}. \tag{3}$$

Under conditions such that the variation $d(x)$ is smooth at the scale of k_F^{-1}, the potential $\epsilon_n(x)$ in (3) is semiclassical. For all values of n which satisfy the condition $n < n_{\max}(kd)$, electron reflection effects are inconsequential, while at $n > n_{\max}(kd)$ there exists a classically forbidden region, and the transmission coefficient is exponentially small. The value of $n_{\max}$ is found from the condition that the semiclassical momentum $p_n(x) = \{2m[E - \epsilon_n(x)]\}^{1/2}$ be real at the narrowest point $(x = 0)$; the result is

$$n_{max}(kd) = [\,kd/\pi\,], \tag{4}$$

where $[X]$ means the greatest integer in X. States corresponding to $n < n_{\max}$, for which the wave function is an oscillatory exponential function, contribute to the current:

$$\psi_n(x) = \sqrt{\frac{p_n(\infty)}{p_n(x)}} \exp\left\{ \frac{i}{\hbar} \int_0^x p_n(x')dx' \right\} . \tag{5}$$

The potential of the electric field applied to the contact varies smoothly in the region of the constriction and assumes constant values $\pm eV/2$ far from the constriction. The current through the contact is conveniently calculated in the region of a constant value of the potential. The conductance $G = dI/dV|_{V=0}$ is determined by the transmission-coefficient matrix T_{nm} of the electron waves corresponding to the various channels.[3] In the case of an adiabatic passage through the constriction, there is no entangling of channels, and we have $T_{nm} = \delta_{nm}\theta(n_{\max} - n)$ The conductance is

$$G(k_F d) = \frac{e^2}{\pi\hbar} n_{max}(k_F d). \tag{6}$$

The sharp, stepped change in $G(k_F d)$ at $k_F d = \pi n$ is a consequence of the semiclassical approximation, which is valid for solutions of Eq. (3). Incorporating tunneling and above-barrier reflection near the classical turning point leads to a spreading of the sharp edge of the steps. A unit change in $n_{\max}$ corresponds to a passage of the turning point through the center of the constriction, where the function $\epsilon_n(x)$ has a maximum. The shape of the step, $\delta G[(k_F d/\pi) - h]$, depends on the curvature at the center of the constriction, $\partial^2 d/\partial x^2 = 2/R$ and is given by

$$\delta G(z) = \frac{e^2}{\pi\hbar}[1 + \exp(-z\pi^2\sqrt{2R/d})]^{-1}, \qquad z = \frac{k_F d}{\pi} - n . \tag{7}$$

It can be seen from (7) that the width of a step depends only weakly on the index n. We wish to call attention to the numerical factor of $\pi^2\sqrt{2}$ in the exponential function in (7). This factor makes the step sharp even in the case $R = d$. Expression (7) gives us the shape of a step at low temperatures: $T < \Delta = n\hbar^2/m(2Rd^3)^{1/3}$ At $T > \Delta$, the factor $\pi^2\sqrt{2R/d}$ in the exponential function in (7) should be replaced by $\hbar^2\pi^2 n/md^2T$.

If the situation is not adiabatic, there will be transitions between channels corresponding to different values of n. This circumstance does not prevent a quantization of the conductance G if the reflection probability is low for this change in channel index. It can be shown that the condition that the reflection probability be small imposes no further restrictions beyond the condition that the steps be sharp, (7):

$$\pi^2\sqrt{2R/d} > 1 , \tag{8}$$

If the length of the constriction, L, exceeds $\sqrt{Rd}$ by a factor of several units, the width of a channel will change over this distance by an amount sufficient to allow the voltage drop to concentrate in the adiabatic region of the motion of the electrons. In this case ($L > \sqrt{Rd}$), the accommodation regions make no substantial contribution to the overall resistance.

An important point for the quantization of the conductance is that the scattering by impurity does not give rise to transitions between adiabatic terms. This condition means that the mean free path l (not to be confused with the transport mean free path

l_{tr}) must be sufficiently large:

$$l \gg \sqrt{Rd}. \qquad (9)$$

A smooth change in the shape of the contact appears to have been arranged in Refs. 1 and 2, by virtue of the electrostatic nature of the potential barriers forming the constriction. The conductance steps are exhibited by samples which have a constriction in the form of a small bridge. For samples with branching in the constriction region,[4] a quantization of G is not observed, possibly because of an unavoidable disruption of the adiabatic situation in the case of a geometry of this sort. A stepped dependence of G on $k_F d$ might also be manifested in contacts of three-dimensional conductors, if the condition for an adiabatic nature were satisfied.

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[2)]In contacts with a sharp geometry, in contrast, the pronounced mismatch of the constriction region and the two-dimensional layer will cause the quantum changes in the conductivity to be manifested as spikes in $G(k_F d)$ corresponding to a resonant transmission of electrons. The pattern of spikes is not universal; it will depend on the aperture reflection of the electron waves.

[1]B. J. van Wees, H. van Houten, C. W. Beenakker *et al.*, Phys. Rev. Lett. **60**, 848 (1988).
[2]D. A. Wharam, T. J. Thornton, R. Newbury *et al.*, J. Phys. **21**, L209 (1988).
[3]J. Imry, in: *Directions in Condensed Matter Physics* (G. Grinstei and G. Masenko, editors), World Scientific Publ., Singapore, 1986, p. 101.
[4]G. Timp *et al.*, Phys. Rev. Lett. **58**, 2814 (1987).

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