Quantum-mechanical conductivity of a ballistic point contact

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The conductivity of a ballistic point contact in the form of a long bridge is calculated. The diffraction of an electron at the transition from the bridge to the massive bank is taken into account. The dependence of the conductivity on the bridge width is described by a smoothed-out staircase with steps of height $e^2/\pi\hbar$.

Experiments[1,2] which have revealed a quantization of the conductivity of a Sharvin point contact in a 2D electron gas of a GaAs/GaAlAs heterojunction have attracted much interest in recent months. The quantization is manifested in the following way: As the width of the contact bridge d is increased, the conductivity G increases not

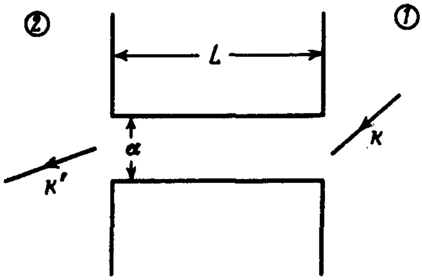

FIG. 1.

monotonically but "up a staircase" with steps of height $e^2/\pi\hbar$. An interpretation of this effect in Refs. 1 and 2 is based on the assumption that the bridge is a long strip (Fig. 1). With increasing d, there is an increase in the number of quantization levels along the d direction which lie below the Fermi level in banks 1 and 2 (the number of conductivity channels, N). The contribution of each channel to G is $e^2/\pi\hbar$, according to Refs. 1 and 2; the channels do not interact.

The steps are most obvious at small values of G, at which N is small, i.e., under the condition $d \sim \lambda_F = 2\pi/k_F$, where k_F is the Fermi momentum. Under these conditions, a calculation of G should allow for the diffraction of the electron wave as it passes from one bank to the other. This circumstance was not considered in Refs. 1 and 2.

According to Ref. 3, the conductivity of a ballistic point contact incorporating diffraction is

$$G = e^2 N_F \langle \theta(\mathbf{v}_\mathbf{k}) \sigma_\mathbf{k} \mathbf{v}_\mathbf{k} \rangle . \qquad (1)$$

Here N_F is the state density at the Fermi surface, and $\sigma_\mathbf{k} = P/I_0$ is the transmission thickness for a plane wave $\mathbf{k}$ which arrives at the point contact from bank 1 (I_0 is the flux density in wave $\mathbf{k}$, and P is the total flux which goes off to bank 2). The angle brackets mean an expectation value over the Fermi surface, and the factor θ restricts the integration to that region in which the electron velocity $\mathbf{v}_\mathbf{k}$ is directed toward the point contact. Expression (1) can be found from the results of Ref. 4.

When the point contact is a long waveguide ($L \gg d$), $\sigma_\mathbf{k}$ can be expressed in terms of quantities which describe diffraction by the end of a semi-infinite waveguide. In the case of a single conductivity channel ($N = 1$) we find

$$\sigma_\mathbf{k} = \frac{|A_\mathbf{k}|^2}{|1 - R^2 e^{2iqL}|^2} \int d\sigma_{\mathbf{k}'} |T_{\mathbf{k}'}|^2 . \qquad (2)$$

Here $2\pi/q$ is the length of the waveguide wave, R is the reflection coefficient for this wave at the open end of the waveguide, $T_{\mathbf{k}'}$ is the amplitude of wave $\mathbf{k}'$, which is emitted by a waveguide wave of unit amplitude, and $A_\mathbf{k}$ is the amplitude of the waveguide wave excited by wave $\mathbf{k}$, of unit amplitude. Substituting (2) into (1), expressing $A_\mathbf{k}$ in terms of $T_\mathbf{k}$ on the basis of the reciprocity principle, and expressing the integral of $|T_k|^2$ over angles in terms of 1-$|R|^2$ on the basis of energy conservation, we find

$$G = \frac{e^2}{\pi \hbar} \frac{(1-|R|^2)^2}{|1-R^2 e^{2iqL}|^2} \equiv \frac{e^2}{\pi \hbar} g . \tag{3}$$

Expression (3) is valid, regardless of whether the waveguide is two-dimensional or three-dimensional. Setting $R=|R|e^{i\theta}$ and $1-|R|^2=|T|^2$, we find

$$g = \frac{|T|^4}{4(1-|T|^2)\sin^2\varphi + |T|^4}, \quad \varphi = \theta + qL . \tag{4}$$

We thus see that we have $g \leqslant 1$. We would have $g=1$ in two cases: that in which there is no diffraction by the open end of the waveguide ($R=0$) and that of resonant transmission, in which the phase shift along the length of the waveguide is cancelled by the phase of the reflection coefficient ($\sin\varphi = 0$).

For the 2D geometry in Fig. 1 we have $N=1$ if $1/2 < d/\lambda_F < 1$. In this case we have $q^2 = k_F^2 - (\pi/d)^2$, and the reflection coefficient R can be taken from the solution of the problem of the emission into a half-space of an H_{10} electromagnetic wave in a waveguide with infinitely high walls.[5] At the threshold $d=\lambda_F/2 (q=0)$ we have $|R|^2=1$, but as d is increased further, the value of $|R|^2$ decreases rapidly, so that we have $|R|^2 < 0.01$ as early as $d > 0.7\lambda_F$. Accordingly, there must be a plateau $G=e^2/\pi\hbar$ on the plot of G versus d at a certain distance from the threshold, and this is indeed what we observe. On the other hand, the d dependence of G near the threshold should be described by a sequence of narrow resonant peaks, but these peaks are not observed.

A possible reason for the disappearance of these peaks might be a thermal broadening of the Fermi surface. If $T \gg \hbar v_F/L$, then we have

$$G = \frac{e^2}{\pi \hbar} \frac{1-|R|^2}{1+|R|^2} \equiv \frac{e^2}{\pi \hbar} \bar{g} , \tag{5}$$

corresponding to a gradual transition to a plateau. It is important to note that $\bar{g}$ depends only weakly on the particular way in which the waveguide is joined to the bank, as can be verified by taking R from the problem of the emission of an H_{10} wave into an unbounded space[6] to calculate $\bar{g}$, for comparison.

The generalization of (2) to the case $N > 1$ is trivial, although very laborious. It can be seen from the resulting expression that the relation $G \neq (e^2/\pi\hbar)N$ generally holds, since the conductivity channels interact (because the waves of different types convert into each other upon reflection from the end of the waveguide), and the conductivity of each channel is not equal to $e^2/\pi\hbar$. These effects, however, are important only near the threshold for the appearance of new waveguide waves. For example, in the width interval $1 < d/\lambda_F < 3/2$, in addition to the fundamental $n=1$ wave, which is symmetric with respect to the waveguide axis, an antisymmetric $n=2$ wave will propagate. Because of the difference in symmetry, the $n=1$ and $n=2$ waves will not convert into each other upon reflection from the end of the waveguide; i.e., these two channels contribute additively to the conductivity: $G = G_1 + G_2$. Here G_1 and G_2 are found from an expression like (3) or (5). It can be seen from Ref. 6 that in the interval under consideration here the reflection coefficient satisfies $|R_{11}|^2 < 10^{-3}$, so we have $G_1 \approx e^2/\pi\hbar$. With regard to R_{22}, we find $|R_{22}|^2 = 1$ at the threshold $d = \lambda_F$, but beyond this point it falls off rapidly. Even at $d > 1.25\lambda_F$ we have $|R_{22}|^2 < 10^{-2}$. In other words, G_2 increases from zero and reaches a plateau $G_2 \approx e^2/\pi\hbar$. In the width interval $3/2 < d/\lambda_F < 2$, an $n = 3$ symmetric wave appears. Upon reflection from the end of the waveguide, the $n = 1$ and $n = 3$ waves convert into each other, so the contributions of these two channels to G are not additive. As early as $d > 1.75\lambda_F$, however, the reflection and conversion coefficients are less than 1%, so G reaches a plateau $3(e^2/\pi\hbar)$.

Under the experimental conditions of Refs. 1 and 2, the value $\lambda_F \approx 400$ Å would have prevailed. The quantity L is more difficult to estimate, since the profile of the potential which is created by the gate and which determines the actual shape of the bridge is not known. In any case, L would not exceed the lithographic dimensions of the gap in the gate, so we would have $L \lesssim 1$ μm. The mean free path with respect to elastic scattering estimated from the mobility is about 10 μm in the better samples, so the point contacts are indeed ballistic. At $L \lesssim 1$ μm and $T \lesssim 1$ K, however, the thermal broadening of the Fermi level would not be sufficient to wash out the resonant peaks. Another possible reason for this washing out might be a variation in d along the length of the waveguide.

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