Magnetic properties of type-II superconductors in the fluctuation region

E. B. Kolomeĭskiĭ
A. V. Shubnikov Institute of Crystallography, Academy of Sciences of the USSR

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Regardless of the nature of the phase transition, type-II superconductivity is replaced by type-I superconductivity inside the fluctuation region as the phase-transition point is approached.

The nature of the phase transition to the normal state in type-II superconductors is still being discussed. In a widely known paper[1] it was concluded on the basis of the 4-ϵ expansion that this phase transition is a discontinuous transition. The opposite conclusion was reached in Ref. 2 on the basis of a numerical analysis of a 3D lattice model and also several rather convincing analytic arguments. The reason for this contradiction is not clear at this time.

We have focused attention here on highly nontrivial magnetic properties of type-II superconductors which manifest themselves in a fairly narrow neighborhood of the transition temperature T_c, regardless of the nature of the transition. The magnetic properties of superconductors depend on the relationship between the London penetration depth δ and the correlation length ξ. As the phase transition point is approached, both these lengths increase without restriction. According to the Ginzburg-Landau theory, they increase in accordance with the same law, so the expression describing their relationship $\kappa = \delta/\xi$ is a constant on the order of the relationship between these quantities at absolute zero.[3] The magnetic properties of the superconductors (type I or type II) therefore do not undergo qualitative changes upon approaching the phase transition temperature if the critical fluctuations are ignored. We will show below that δ increases much slower than ξ in the fluctuation region. These quantities therefore become equal in order of magnitude near the phase transition temperature and the superconductor becomes a type-I superconductor even if it was a type-II superconductor in the region in which the Ginzburg-Landau theory applies.

Let us consider, for example, a clearly type-II superconductor. In the temperature region where $\ln(\delta/\xi) \gg 1$ the expression for the linear energy of the Abrikosov vortex is

$$\epsilon = \left(\frac{\Phi_0}{4\pi\delta}\right)^2 \ln\left(\frac{\delta}{\xi}\right),$$

where $\Phi_0 = \pi\hbar c/e$ is the flux quantum. Let us extrapolate this flux quantum to the fluctuation region and assume that ϵ and δ depend on ξ. According to the scale-invariance hypothesis,[4] the free energy of the linear object ϵ vanishes as T_c/ξ in this region. It thus follows that

$$\delta^2 \sim \xi \frac{\Phi_0^2}{T_c} \ln\left(\frac{\delta}{\xi}\right). \tag{1}$$

The use of the expression for the vortex energy is not the only way the functional dependence $\delta(\xi)$ can be obtained (more on this below): It is easier for us to begin with an equation which contains the electron charge. Relation (1) shows that δ increases primarily as $\xi^{1/2}$. Consequently, if ξ is large enough, the penetration depth is equal in order of magnitude to the correlation length and the superconductor becomes a type-I superconductor. The correlation length ξ^* at which this situation occurs can be estimated from the relation

$$\xi^* \sim \Phi_0^2/T_c.$$

In deriving Eq. (1) we assumed that we are inside the fluctuation region. The correlation length ξ^* which we found must therefore be greater than that length scale beyond which, according to the Levanyuk-Ginzburg criterion,[4] the Landau theory breaks down. It can be shown that this condition is equal to the requirement that the superconductor must be, according to the Ginzburg-Landau theory, a type-II superconductor. Expressing ξ^* in terms of κ (outside the fluctuation region) and the Ginzburg number Gi in the BCS theory,[3] we find

$$\xi^* \sim \frac{\hbar}{p_F} \kappa^2\, \mathrm{Gi}^{-3/4}, \tag{2}$$

where P_F is the Fermi momentum.

For $\xi > \xi^*$ the dependence $\delta(\xi)$ differs from (1) because of the absence of a logarithm. Making use of the evaluation of δ (Ref. 3): $\delta^2 = mc^2/4\pi ne^2$, where m is the electron mass and n is the density of the superconducting electrons, we obtain the relation

$$n \sim m T_c \hbar^{-2} \xi^{-1},$$

in which there is no electron charge. This relation was derived by Josephson[5] for the density of superfluid helium near the λ point. In the case of a charged superfluid liquid, this relation immediately shows that all superconductors which are in reasonable proximity of the transition point are type-I superconductors.

Let us determine the temperature region ΔT in which type-I superconductivity exists. In the fluctuation region we have

$$\xi \sim \frac{\hbar}{p_F} |(T-T_c)/T_c|^{-\nu}\,\mathrm{Gi}^{-1/4}\,.$$

Ignoring the small value of the heat capacity, we find for ν, within a very small error, the value $\nu = 2/3$ (Refs. 3 and 5). Substituting this value into Eq. (2), we find

$$\Delta T \sim T_c \kappa^{-3}\,\mathrm{Gi}^{3/4}\,. \tag{3}$$

According to Halperin *et al.*,[1] the "size" of the transition region for type-I superconductors, ΔT_{I}, is

$$\Delta T_{\mathrm{I}} \sim T_c \kappa^{-3}\,\mathrm{Gi}\,. \tag{4}$$

At first glance, it may seem that the transition to a type-I superconductor is simply the consequence of a different interpretation of the results of Ref. 1 and that the difference between the values 3/4 and 1 is the result of a temperature dependence that has been ignored. Expression (3) was derived from the Josephson relation, which takes into account the fluctuations of the order parameter, and Eq. (4) was obtained by additionally taking into account the fluctuations of the vector potential. The latter addition, however, would introduce the temperature dependence of Φ_0, which obviously cannot be done.[6] Relation (3) is therefore intrinsically different from relation (4). The interval ΔT can belong to the fluctuation region only if the inequality $\kappa \gg \mathrm{Gi}^{-1/12}$ holds.

Regardless of the nature of the transition to the normal state, the "magnetic field-temperature" phase diagram therefore must have a point at which the curves of the upper critical field merge with the curves of the lower critical field. Since there is a correlation length ξ^* where the superconductivity changes its nature, we conclude that the critical behavior of a granular superconductor, whose grain size is no greater than ξ^*, is different from that of a bulk single crystal of the same substance (the characteristic value of ξ^* ranges from several microns to several centimeters).

It is hoped that it would be easier to detect the predicted effects in the high-T_c superconductors, for which the Ginzburg number is, according to the estimates of Refs. 7 and 8, moderately small: $\mathrm{Gi} \sim 10^{-5}$–10^{-2}.

Because of the known similarity between superconductors and smectics-A (Ref. 9), all our conclusions apply to smectics-A.

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