Formation of dense emitting plasma bands (marfes)

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A two-dimensional energy-balance equation for the plasma in a tokamak is analyzed. Radiative loss is taken into account. The formation of marfes may be thought of as a bifurcation of the solutions of the energy-balance equation.

Experiments at many tokamaks at plasma densities near the critical value, where radiative energy loss plays a major role, have revealed the formation of dense, relatively cool, and intensely emitting bands of plasma called "marfes,"[1] which usually localize near the inner boundary of the plasma column. The lifetime of the marfes in the TFTR and the JET reaches several seconds,[2] demonstrating the stability of these poloidally asymmetric structures. Lipschultz *et al.*[1] have suggested that marfes may originate from a nonmonotonic dependence of the power of the impurity radiative loss on the electron temperature T, as in the corona approximation.[3] Analysis of the linear stage of a radiative instability[4,5] has revealed that poloidally asymmetric modes may develop at a growth rate higher than that for a poloidally symmetric mode. In the present letter we retain the radiative model for an analysis of the nonlinear stage of the onset of marfes, working from a two-dimensional nonlinear energy-balance equation for the plasma.

We assume that the thermal conductivity of the plasma across the magnetic field is given by $\kappa_\perp = \kappa_\perp T^{(\beta-1)}/\beta$, while that along the magnetic field is described by the classical expression $\kappa_\parallel = (2/7)\kappa_\parallel T^{5/2}$, where β is an adjustable parameter, and $\kappa_\perp$ and $\kappa_\parallel$ are normalization constants. Ignoring the convection terms, we write the energy-balance equation for the plasma at the periphery of the column, $r \sim a$ (a is the minor radius of the tokamak), where the marfes usually arise, as follows:

$$\hat{\kappa}_\perp \frac{\partial^2 \theta}{\partial x^2} + \frac{\hat{\kappa}_\parallel}{(qR)^2} \frac{\partial^2 \theta^\alpha}{\partial \varphi^2} - Q(\theta) = .$$

Here $\theta = T^\beta$; $x = a - r$; φ is the poloidal angle; q is the safety factor; R is the major radius of the tokamak; and the function $Q(\theta)$ models the T dependence of the radiative loss.

We are interested in asymmetric solutions of (1) which are not stimulated by an asymmetry of the boundary conditions at $x = 0$ or $x \to \infty$ (the center of the plasma column). We accordingly restrict the discussion to the two-mode approximation, $\theta(x,\varphi) = \theta_0(x) + \theta_1(x)\sin\varphi$, adopting the following as boundary conditions on (1):

$$\hat{\kappa}_\perp (d\theta_0/dx)|_{x=0} = W_L \equiv G(\theta_0(0)), \quad (2)$$

$$\hat{\kappa}_\perp (d\theta_0/dx)|_{x\to\infty} = W_\infty, \quad (3)$$

$$\theta_1(x=0) = \theta_1(x\to\infty) = 0. \quad (4)$$

Here W_∞ is the energy flux density directed away from the central part of the column to the periphery; W_L is the energy flux density to the scrape-off layer; and $G(\theta)$ shows the dependence of W_L on $\theta_0(0)$, which can be found from the solution of the energy-balance equation for the scrape-off layer. We will not need the explicit expression for $G(\theta)$, and we will restrict the discussion to qualitative arguments, assuming that $G(\theta)$ is a monotonically increasing function of θ.

We consider poloidally symmetric solutions of (1). From Eqs. (1)–(3) we find an equation for $\theta_L = \theta_0(x=0)$:

$$G(\theta_L) = [\, W_\infty^2 - 2\hat{\kappa}_\perp \int_{\theta_L}^{\infty} Q(\theta)\, d\theta \,]^{1/2}, \tag{5}$$

under the assumption that $Q(\theta)$ falls off rapidly with increasing θ.

It is not difficult to see from (5) that for a nonmonotonic functional dependence $Q(\theta)$ of the type $Q(\theta) = Q_*(1-(1-\theta/\theta_*)^2)$, where Q_* and θ_* are characteristic parameters of the function $Q(\theta)$, and at a low impurity density n_I, with $Q \sim n_I$ and $f_0 = 2\kappa_\perp \int_0^\infty Q(\theta) d\theta / W_\infty^2 < 1$, two stable solutions (5) can exist. The two solutions correspond to "weak" and "strong" radiative losses. The transition between these states as n_I is increased may be abrupt, in accordance with the results of the linear analysis of the stability of the energy balance at the periphery of the plasma column which was carried out by Ohyabu.[6] In the case $f_0 > 1$, there may be no more than a single stable solution (5); it would correspond to a weak radiative loss.

We now consider poloidally asymmetric solutions of (1). Taking an average of (1) over the poloidal angle with weights of 1 and $\sin\varphi$ for $\alpha = 1$ and 2, and assuming $(\theta) = Q_* \delta(\theta - \theta_*)$ [$\delta(y)$ is the Dirac δ-function], we find

$$\frac{d^2 t_0}{d\xi^2} = \frac{f_0}{2\pi (t_1^2 - t_0^2)^{1/2}} \hat{\theta}(t_1 - t_0),$$

$$\frac{d^2 t_1}{d\xi^2} = \kappa^2 t_1 t_0^{(\alpha-1)} - \frac{f_0 (t_0/t_1)}{\pi (t_1^2 - t_0^2)^{1/2}} \hat{\theta}(t_1 - t_0), \tag{6}$$

where $\hat{\theta}(y) = 1 (y>0), 0 (y<0)$; $t_0 = (\theta_0 - \theta_*)/\theta_*$; $t_1 = \theta_1/\theta_*$; $\xi = x/r_*$; $r_* = \kappa_\perp \theta_*/W_\infty$; $f_0 = 2\kappa_\perp Q_*/W_\infty^2$; and $\kappa^2 = \kappa_\parallel \kappa_\perp \theta_*^2 (2\theta_*)^{\alpha-1}/(qRW_\infty)^2$.

A solution of (6) of the marfe type, with the emitting zone concentrated in a small arc of the poloidal angle, $\Delta\varphi \lesssim 1$ corresponds to the condition that the relative change in t_1 and t_0 is small at $t_1 \gtrsim t_0 \approx t_* > 0$, i.e., the case in which there is a radiative loss in (6). In this case, integrating (6) separately in the ξ regions $t_1 < t_0$ and $t_1 > t_0 \approx t_*$, and using (2)–(4), we find the following system of equations for t_* and $t_L = \theta_L/\theta_* - 1$ ($\alpha = 1$):

$$\frac{3}{2} \frac{1 - g(t_L)}{1 + \kappa t_*} = \frac{1}{2} \frac{f_0}{f_{min}} \ln\left(\frac{f_0 + f_{min}}{f_0 - f_{min}}\right) \geqslant 1, \tag{7}$$

$$\tan^{-1}\left[\frac{\kappa(t_* - t_L)}{g(t_L)}\right] - 1 = \frac{1 + g(t_L)}{\kappa t_*}, \tag{8}$$

where $g = G/W_\infty$ and $f_{\min} = 2\pi(1+\kappa t_*)\kappa t_*/3$.

It follows from (7) that a necessary condition for the existence of solution (7), (8) is $g < 1/3$. The condition $\Delta\varphi \lesssim 1$ holds at $f_0 \lesssim 1$, i.e., if there exists no poloidally symmetric stable solution of (1) with an intense radiative loss, as was shown above. The quantities t_* and t_L themselves are found from the specific functional dependence $g(t)$; their determination goes beyond the scope of the present paper. For $\alpha = 2$, the solution of (6) again leads to equations like (7), (8), but we will not reproduce those lengthy results here. We simply note that the existence of marfe solutions in this case is limited by the conditions $g < 1/3$, $f_0 \gtrsim 1$.

In summary, the model examined in this letter shows that the onset of marfes may be thought of as a bifurcation of the solution of the energy-balance equation which results in the appearance of poloidally asymmetric solutions characterized by a strong radiative loss, comparable to the plasma-heating power.

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