Gravitational instantons and the branching off of the expanding universe

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A model of a gravitational field which interacts with the fields of matter was used to find the instanton which describes the branching off from a flat universe of a little, closed universe which subsequently expands classically under inflationary conditions. The existence of such instantons raises the possibility of self-nucleation of systems of expanding universes.

The possibility that little, closed universes can branch off from the large universe (including a flat universe) has recently been widely discussed.[1–3] These processes, which are connected with the quantum fluctuations of the topology, can lead to intriguing physical consequences,[1–7] such as the loss of quantum coherence, nonlocalizability, the vanishing of the cosmological constant, etc. Classical solutions of Euclidean field equations (gravitational instantons) can be used to describe the branching processes. An explicit example of such instanton was given by Giddings and Strominger.[3] Its metric has an O(4) invariant form of the type

$$ds^2 = d\tau^2 + a^2(\tau)\, d\Omega_3^2$$

This corresponds to a Euclidean manifold illustrated in Fig. 1a.

We know that an analytic continuation of the given solution, with $\tau = 0$, to the metric with a Lorentzian signature, $\tau \to it$, is a closed universe which classically contracts from the radius $a_0 \equiv a(\tau = 0)$ to $a = 0$. The instanton used by Giddings and Strominger[3] thus describes the branching off of a contracting universe. In this letter we show that some models have gravitational instantons which describe the branching off of an expanding universe and which have the shape illustrated in Fig. 1b (since in the analytic continuation, which is realized at $\dot{a} = 0$, we have $\ddot{a} \to -\ddot{a}$, the manifolds illustrated in Fig. 1, a and b, actually correspond to the branching off of contracting and expanding universes.)

Let us consider a model defined by Euclidean action

$$S = \int d^4x \sqrt{g} \left[-\frac{1}{2\kappa} R + \frac{1}{6} H^2_{\mu\nu\lambda} + \frac{1}{2} (\partial_\mu \varphi)^2 + V(\varphi) \right], \qquad (1)$$

where $H_{\mu\nu\lambda} = \nabla_{[\mu} A_{\nu\lambda]}$ is the strength of the axion field, and the behavior of $V(\varphi)$ is illustrated in Fig. 2a. We need to find a solution in $O(4)$ invariant form as in (1). We assume $\varphi = \varphi(\tau)$, $H_{0ij} = 0$, and $H_{ijk} = N\epsilon_{ijk}$, where N does not depend on τ. We need to solve the field equations

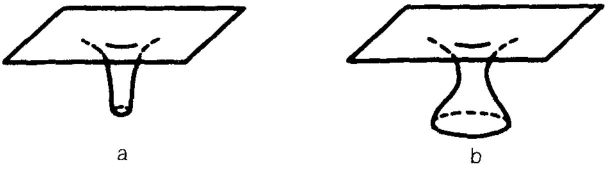

FIG. 1.

$$\ddot{\varphi} + 3\frac{\dot{a}}{a}\dot{\varphi} = \frac{dV}{d\varphi}, \tag{2}$$

$$\dot{a}^2 - 1 = \frac{\kappa}{3}a^2\left(\frac{1}{2}\dot{\varphi}^2 - V(\varphi)\right) - \frac{\kappa}{3}\frac{N^2}{a^4}, \tag{3}$$

$$\left(\frac{\dot{a}}{a}\right)^{\cdot} = -\frac{1}{a^2} + \kappa\frac{N^2}{a^6} - \frac{\kappa}{2}\dot{\varphi}^2, \tag{4}$$

where the boundary conditions $\dot{a}(0) = \dot{\varphi}(0) = 0$; $a(\tau) \to \tau, \varphi(\tau) \to \varphi_\infty$ as $\tau \to \infty$.

Equation (2) formally describes the motion of a particle with the "coordinate" φ in the potential $-V$ (Fig. 2b) with a "viscous friction" $3\dot{a}/a$. The boundary conditions correspond to the motion, with zero initial velocity, from a certain point $\varphi(\tau=0) = \varphi_0$ to the point $\varphi(\tau=\infty) = \varphi_\infty$. Such a motion can occur if the "friction" is negative $(\dot{a}/a < 0)$ at certain values of τ, which is consistent with Fig. 1b.

Let us consider the case $V_1/V_{\max} \sim \kappa V_{\max}(\Delta\tau)^2 \ll 1$, $\kappa^{3/2}V_1 \ll N^{-1}$, where $\Delta\tau$ is the time scale for the sliding without friction. The evolution of $a(\tau)$ and $\varphi(\tau)$ would then occur in two stages: φ first would vary between $\varphi_0 \approx \varphi_1$ and the values close to φ_∞, with $a = a_0 \approx \sqrt{3/\kappa V_1}$, remaining virtually constant. The function $a(\tau)$ would then vary, tending to the asymptotic limit $a(\tau \to \infty) = \tau$, with $\varphi \approx \varphi_\infty$ remaining constant. It follows from (3) and (4) that at the first stage we would have $\dot{a}/a \approx -\kappa/2\int_0^\tau \dot{\varphi}^2 d\tau$ and a would vary negligibly: $\Delta a/a_0 \sim \kappa V_{\max}(\Delta\tau)^2$. The work done by the (negative) frictional force in this case is estimated to be $A = -3\int \frac{\dot{a}}{a}\dot{\varphi}^2 d\tau \sim \kappa V_{\max}^2(\Delta\tau)^2$, which is sufficient to reach the point $\varphi = \varphi_\infty$. If

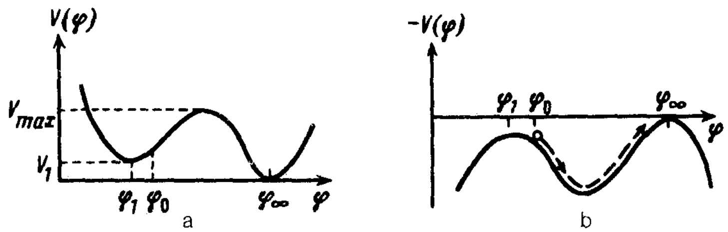

FIG. 2.

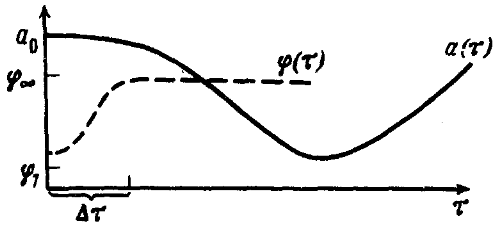

FIG. 3.

the value of τ is large (at the second stage), the effect of the field φ can be ignored and the solution will match the Giddings and Strominger's[3] instanton, for which $a(\tau\to\infty)=\tau$. The solution is shown schematically in Fig. 3.

In the approximation under consideration the Euclidean action for the instanton which we found is

$$S=2\pi^2 a_0^3 \int_{\varphi_0}^{\varphi_\infty} d\varphi\,\sqrt{2V}\,(1+O\,(NV_1\kappa^{3/2}))\,.$$

We note that $S\gg 1$, suggesting that the semiclassical approximation can be used. We should also note that instanton parameters are different from Planckian parameters by a wide margin.

Analytic continuation (at $\tau=0$) of the described solution to the Lorentz signature corresponds to the classical expansion of a closed universe filled with a scalar field that oscillates near $\varphi=\varphi_1$. The oscillations decay with the passage of time and the expansion reaches the stage of inflation with the Hubble constant $\sqrt{\kappa V_1/3}$.

The existence of instantons which describe the branching off of expanding universes raises the possibility of self-nucleation of a universe or a system of universes, as illustrated in Fig. 4. This possibility stems from the fact that the concept of time is not understood in the framework of quantum gravitation in the nonclassical region. Such a model can conceivably be used to dynamically determine the coupling constants in a self-consistent way and to explain the vanishing of the cosmological constant.

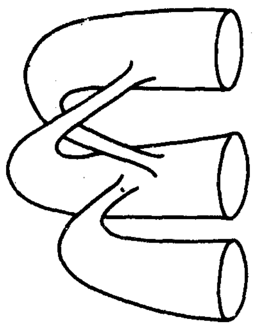

FIG. 4.

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