Photon showers and the Belkacem peak in the radiation from channeled ultrarelativistic electrons

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(Submitted 1 August 1988)
Pis'ma Zh. Eksp. Teor. Fiz. **48**, No. 6, 303–306 (25 September 1988)

The solution of cascade integrodifferential equation is analyzed and the particular features of the kinetics of radiative stopping of ultrarelativistic channeled electrons are identified. This approach is used to develop a theory to explain the Belkacem peak which has been detected in the hard part of the radiated-energy spectrum.

1.. Experimental studies of electrons with energies $E_0 = 150$ GeV, carried out recently[1,2] in CERN, have revealed that the intensity of the hard part of the spectrum of the emitted energy increases appreciably as the angle between the direction of the incident beam and the $\langle 110 \rangle$ axis of a germanium single crystal decreases (the Belkacem peak). Attempts to explain the origin of this peak in terms of radiation cooling of the beam[3,4] left unanswered the question of the dependence of the intensity of the cascade processes on the transverse energy and could not reconcile theory with experiment from the standpoint of the description of the spectrum shape or from the standpoint of the multiplicity of the emitted γ rays.

We will show that the characteristic feature of the cascade processes peculiar to ultrahigh-energy electrons is the fact that the particle-distribution maximum does not move along the energy axis, broadening simultaneously as it does in the case of electron stopping in a substance.[5] Instead, this maximum forms in the region of large energy loss, $\Delta E \sim E_0$, as it passes a layer of the substance of thickness on the order of the effective radiation length. This circumstance determines to a large extent the properties of the Belkacem peak, relegating the radiation cooling and multiple scattering to a secondary role.

2. The cascade equation describing the evolution of the distribution ρ of the radiative electrons with a thickness l of the crystal can be written in the form

$$\frac{\partial \rho_\epsilon}{\partial l} = \int_0^\infty W_\epsilon\,(E + \hbar\omega,\ \hbar\omega)\,\rho_\epsilon(E + \hbar\omega)\,d\omega - \int_0^E W_\epsilon(E, \hbar\omega)\rho_\epsilon\,(E)\,d\omega\,, \tag{1}$$

where $W_\epsilon\ (E, \hbar\omega)$ is the probability for the emission of a γ ray with a frequency ω by an electron with a total energy E and with the energy ϵ of the transverse motion in an effective potential $V_{\text{eff}}\ (\mathbf{r}_\perp)$ of the axial channel of the crystal. In the magnetic bremsstrahlung limit we have[6]

$$W_\epsilon(E, \hbar\omega) = \frac{W_0}{E^2\,(1+u)}\{\,[\,1 + (1+u)^2]\overline{K_{2/3}(x)} - (1+u)\,\overline{\int_x^\infty K_{1/3}(y)\,dy}\,\}\,. \tag{2}$$

Here

$$u = \frac{\hbar\,\omega}{E - \hbar\omega}\,,\ x = \frac{2u}{3\chi}\,,\ \ \chi = \frac{e\,\hbar\,E}{m^3\,c^5}\left|\,\frac{\partial V_{\mathit{eff}}}{\partial \mathbf{r}_\perp}\,\right|,\ \ W_0 = \frac{(emc)^2}{\hbar\pi\sqrt{3}}\,. \tag{3}$$

The bar over the expression means averaging over the accessible range of the transverse coordinates.

Equation (1), with the integrated kernel (2), was solved on a computer. We will briefly discuss some of the new theoretical results obtained for the ⟨110⟩ axial channeling of electrons with $E_0 = 150$ GeV in germanium at a temperature $T = 100$ K.

3. A striking difference in the kinetics of radiation damping and the kinetics of Landau damping[5] by electrons of the substance is illustrated in Fig. 1 on the basis of the curves for the evolution of the energy distribution, $\rho_\epsilon\ (E)$, for two characteristic values of the transverse energy ϵ. We see that the distribution maximum forms at a sufficiently large energy loss $\Delta E \sim E_0$ as it passes through a layer of the crystal of thickness on the order of the effective radiation length, which depends on the radius R_ϵ of the accessible region. This circumstance reflects the important role in the kinetics of radiation damping played by the emission of hard γ rays, whose energy is comparable with the energy of the radiating electrons. The role of such processes increases with decreasing R_ϵ as a result of the increase in the effective forces which act on the electron in the central part of the crystal channel.

In this connection, we should point out that the case of the Bethe-Heitler radiation in an amorphous medium, in which the relative probability for the emission of hard γ rays is high, physically attracts special attention. As our analysis shows, the peak of the radiating-particle distribution in this case forms at the boundary of the spectrum $\Delta E = E_0$ and is rigidly "bound" to this point.

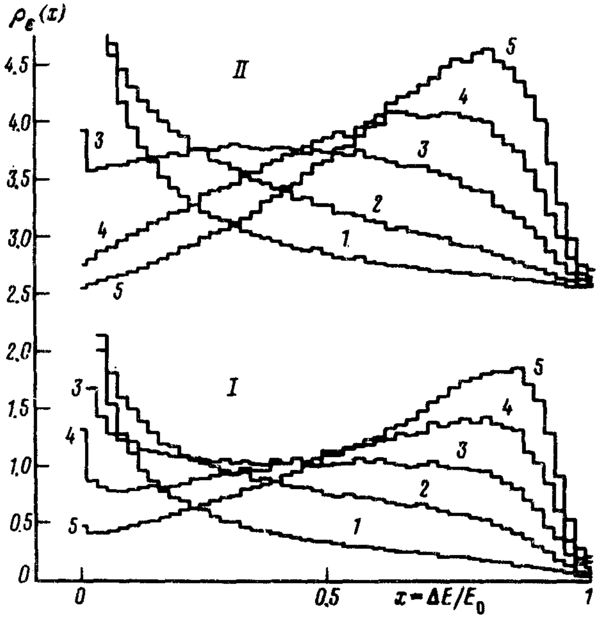

FIG. 1. Evolution of the energy distribution of electrons with $E_0 = 150$ GeV at various transverse-energy levels in the ⟨110⟩ axial Ge channel. *I*—$R_\epsilon/a = 2$: 1—$l = 0.01$ mm, 2—$l = 0.03$ mm, 3—$l = 0.05$ mm, 4—$l = 0.07$ mm, 5—$l = 0.09$ mm; II—$R_\epsilon/a = 5$: 1—$l = 0.05$ mm, 2—$l = 0.125$ mm, 3—$l = 0.25$ mm, 4—$l = 0.375$ mm, 5—$l = 0.5$ mm. Curves II are shifted by a factor of 2.5 up the scale.

4. Solution of the problem on the evolution of ρ_ϵ (E) and knowledge of the initial transverse energy level population make it possible to calculate the spectra of the radiated energy as a function of the parameters of the incident beam.

Let us first consider the angles of incidence larger than the critical angle $(\theta > \theta_c = 6.6 \times 10^{-5}$ rad), when there is only an above-the-barrier fraction of electrons. Making use of the Molière approximation for $V_{\text{eff}}(\mathbf{r}_\perp)$, we were able to reconcile theory with experiment (see Fig. 2) by reducing the constant W_0 by 19% compared with the theoretical value in (3). This shows that the accuracy is of the level which applies, under the conditions being considered, to the magnetic-bremsstrahlung limit (2) and to the approximation of the uniform distribution of the radiating particles in the accessible region of the transverse coordinates.

The value of W_0 which we found was used in analyzing the emission spectra under channeling conditions $(\theta < \theta_c)$. We found that the principal component of the Belkacem peak comes from a very small group of electrons which are captured in the lowest-lying levels ϵ with $R_\epsilon \lesssim 2.5a$ (a is the atomic screening length for Ge). The position of the peak in this case was shifted slightly toward higher energy loss compared with the experimental result.

Clearly, the strong multiple scattering in the central part of the axial channel causes the electrons to escape from those levels which are characterized by small

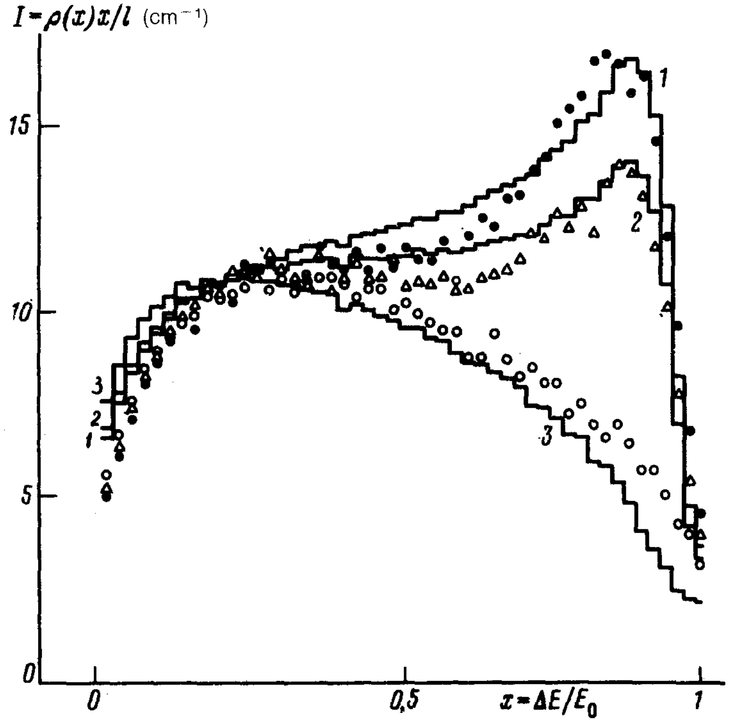

FIG. 2. Spectra of the radiated energy of Ge ($l = 0.185$ mm, $T = 100$ K) for various directions of the incident electron beam ($E_0 = 150$ GeV, $F = 37$ μrad). 1—In the direction of the ⟨110⟩ axis; 2—$\theta = 17$ μrad; 3—$\theta = 96$ μrad. The experimental points were taken from Ref. 1.

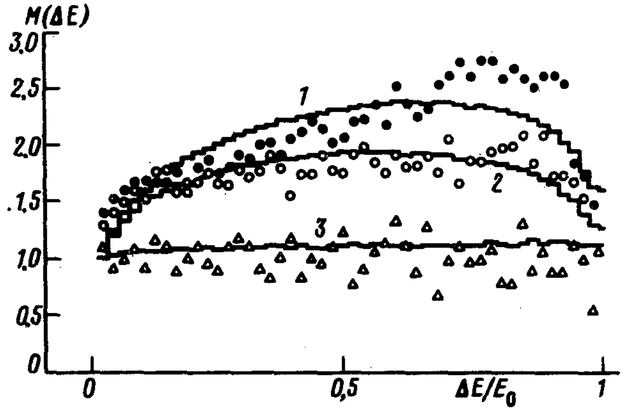

FIG. 3. Multiplicity of the γ rays with $\hbar\omega \geqslant 3$ GeV, plotted as a function of the radiated energy. 1—The electron beam is parallel to the ⟨110⟩ axis; 2—$\theta = 96$ μrad; 3—disoriented single crystal. The parameters of the beam and the crystal correspond to those in Fig. 2. The experimental points were taken from Ref. 2.

values of R_ϵ . We took these "cosmetic" effects into account in a very simple way: by setting the population of levels with $R_\epsilon/a < 1.5$ to zero and, accordingly, increasing the population in the region $1.5 \leqslant R_\epsilon/a \leqslant 2.5$ (this procedure does not depend on the angle of incidence) in such a way that the total integrated population would remain constant. As a result, we obtained a remarkable agreement with the experiment, shown in Fig. 2. An important point is that the structure of the Belkacem peak depends only slightly on the particular features of the population redistribution in the indicated region R_ϵ .

5. The solution algorithm for cascade equation (1) allowed us to simultaneously calculate the average number of emitted γ rays (the multiplicity) $M(\Delta E)$ as a function of the radiated energy ΔE (Fig. 3). The additional background with a Bethe-Heitler spectrum, which was simulated in the calculations in accordance with the experimental conditions,[1,2] has a strong effect on the behavior (decrease) of the multiplicity near the boundary of the spectrum, $\Delta E = E_0$. Upon removal of the background $M(\Delta E)$ will increase monotonically with increasing ΔE.

6. The height $I_{\max}$ of the Belkacem peak and the maximum value $M_{\max}$ of the multiplicity of γ rays, of course, depend strongly on the divergence of the incident beam. At $\theta = 0$ and $l = 0.185$ mm, for example, a decrease in the total width F at half-maximum from 3.7×10^{-5} rad to 1.9×10^{-5} rad in the case of a Gaussian beam causes $I_{\max}$ to increase by a factor of 1.43. $M_{\max}$ in this case increases to 2.72. In the case of a nondiverging beam, with $F = 0$, we found $I_{\max} = 30.8 \text{ cm}^{-1}$ and $M_{\max} = 3.15$.

It can be shown that as the thickness l of Ge single crystal is decreased, the intensity of the Belkacem peak decreases sharply and its position shifts slightly down the energy scale ΔE. This conclusion is a consequence of the characteristic features of the kinetics of radiation damping described above. The corresponding quantitative results will be published in another paper.

We wish to thank V. N. Baĭer for useful discussions.

[1] A. Belkacem *et al.*, Phys. Lett. **B177**, 211 (1986).
[2] A. Belkacem, Thése Docteur-es-Science Physique, Universite de Lyon, 1986.
[3] V. V. Tikhomirov, Phys. Lett. **A125**, 411 (1987).
[4] X. Artru, Phys. Lett. **A128**, 302 (1988).
[5] L. D. Landau, J. Phys. USSR **8**, 201 (1944).
[6] V. N. Baĭer, V. M. Katkov, and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. **92**, 1228 (1987) [Sov. Phys. JETP **65**, 686 (1987)].

Translated by S. J. Amoretty