QCD sum rules and nuclear matter

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(Submitted 14 August 1988)
Pis'ma Zh. Eksp. Teor. Fiz. **48**, No. 6, 307–310 (25 September 1988)

The basic properties of nuclear matter are determined on the basis of the QCD sum rules. The equilibrium density and the binding energy are determined. The expansion of the nucleon in the nucleus is estimated.

Formulation of a systematic theory of nuclear matter is one of the principal goals of nuclear physics. Many studies[1] were based on nucleon-nucleon interaction and therefore required the development of phenomenological representations. These models were introduced long before the appearance of QCD. Since QCD is regarded as the true theory of strong interactions, it would be desirable to have a model of nuclear matter based on QCD.

In this letter we will attempt to determine the basic properties of nuclear matter on the basis of QCD sum rules.[2] Since this method takes into account, though in a very sketchy way, the quark and gluon confinement, it is preferred over other QCD-based methods that describe nuclear matter. Several properties of nucleons ranging from static characteristics to deep inelastic scattering are described on the basis of this method.

The sum rules in a vacuum are, as we know, the dispersion relations for the polarization operator $\Pi_0(q^2)$:

$$\Pi_0(q^2) = i\int d^4y e^{-iqy} \langle 0|T\{\bar{j}(y)j(0)\}|0\rangle, \tag{1}$$

where the hadron (proton) current is[3]

$$j(y) = u^a(y)C\gamma_\mu u^b(y)\gamma_5\gamma_\tau d^c(y)\epsilon^{abc}g^{\mu\tau}. \tag{2}$$

Here u and d are the quark fields, and C is the charge-conjugation operator. In the dispersion relation the nucleon-pole component is clearly identifiable and the remaining states are approximated by a continuum which begins at a certain point W^2

$$\Pi_0(q^2) = \frac{\lambda^2}{q^2-m^2} + \frac{1}{2\pi i}\int_{W^2} \frac{\Delta_{Q^2}\Pi(Q^2)dQ^2}{Q^2-q^2}. \tag{3}$$

The operator $\Pi_0(q^2)$ was calculated by QCD methods, with allowance for the first few terms of the expansion in powers of q^{-2}. After a Borel transformation,[3] the sum rules become a system of two equations for two tensor structures, $\hat{q}$ and I, in Π_0. Ioffe and Smilga[3] showed that the difference between the left side and the right side of Eq. (3) is minimized when

$$m = 1 \text{ GeV}, \lambda^2 = 2.1 \text{ GeV}^6, W^2 = 2.3 \text{ GeV}^2 \tag{4}$$

in the stability interval

$$0.8 \text{ GeV}^2 \leqslant M^2 \leqslant 1.4 \text{ GeV}^2. \tag{5}$$

Let us determine how the quantities in (4) vary in nuclear matter. For this purpose, we construct polarized current operator (2) in nuclear matter

$$\Pi_m(q) = i \int d^4 y e^{-iqy} \langle \mathcal{M} | T\{\bar{j}(y) j(0)\} | \mathcal{M} \rangle \tag{6}$$

and we write the dispersion relations for the difference $\Pi_m - \Pi_0$. Assuming that stability interval (5) is the same for nuclear matter and vacuum, we find equations that describe the variation of the quantities Δ_m, $\Delta\lambda^2$, and ΔW^2 in (4).

The first quantity in (4) is related to the potential energy of the nucleon by the relation

$$U = \frac{\Delta m}{1 + \frac{T}{m}}, \tag{7}$$

where $T = p_F^2/2m$ is the kinetic energy, $p_F = (3\pi^2\rho/2)^{1/3}$ is the Fermi momentum, and ρ is the nucleon density. The quantity λ^2 is proportional to the quark wave function at the origin. Accordingly, $\Delta\lambda^2$ gives us an estimate of the variation of the nucleon radius in the medium.

The polarization operator in medium (6) depends on q^2 and on the component q_0. The latter is determined on condition that the nucleon corresponding to the pole in the dispersion relation belong to the nuclear matter. Ignoring the motion of nucleons, we see that this corresponds to the condition

$$s_A = (A+1)^2 m^2, \tag{8}$$

where A is the number of nucleons in the medium.

The function $\Pi(\dot{q})$ now has, in addition to the singularities in q^2, the specific singularities in the variable $u_A = 2q^2 + 2M_A^2 - s_A$, where M_A is the mass of the bound particle which is comprised of A nucleons. In a bound system in which the binding energy ϵ is larger than the separation energy τ, which nuclear matter is perceived to be, the singularities in u_A arise when

$$q^2 \geqslant m^2 + 2Am(\epsilon - \tau), \qquad -\epsilon - \tau > 0. \tag{9}$$

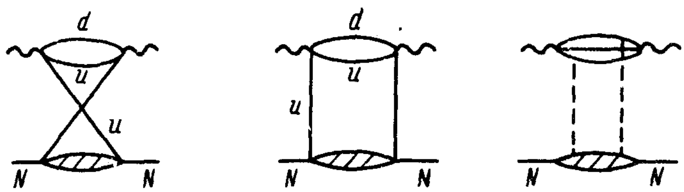

FIG. 1. Diagrams used in the calculation of the structure proportional to $\hat{q}$ in the polarization operator.

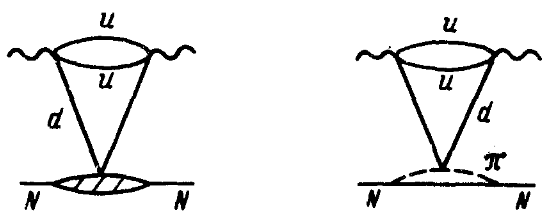

FIG. 2. Diagrams used in the calculation of the structure proportional to $\hat{q}$ in the polarization operator.

The contribution from these singularities is exponentially small after the Borel transformation.

Accordingly, the dispersion relation for $\Pi_m - \Pi_0$ has the form (3). Taking into account on the right side of (3) only the terms which are linear in Δm, $\Delta\lambda^2$, and ΔW^2, we obtain a system of equations for two tensor structures $\hat{q}$ and 1

$$4\pi[3m^2E_0(M^2) + (s-4m^2)E_0(M^2) - M^2E_1(M^2)]\langle M|\bar{u}\gamma_0 u|M\rangle\frac{1}{L} - \pi^2\langle M|-\frac{\alpha_s}{\pi}G^2|M\rangle\frac{1}{L} = \left(-\frac{2m}{M^2}\lambda^2\Delta m - \Delta\lambda^2\right)e^{-m^2/M^2} - \frac{W^4}{2}e^{-W^2/M^2}\frac{\Delta W^2}{L} \qquad (10)$$

$$-2\pi M^4\langle M|\bar{d}d|M\rangle E_1 = \left(1-\frac{2m^2}{M^2}\right)m\lambda^2 e^{-m^2/M^2}\Delta m + m e^{-m^2/M^2}\Delta\lambda^2 + 2aW^2e^{-W^2/M^2}\Delta W^2(-1); \qquad a = 0.55\ \mathrm{GeV}^4,$$

$$E_0(M^2) = 1 - \exp(-W^2(M^2); \; E_1 = 1 - \left(1+\frac{W^2}{M^2}\right)e^{-(W^2/M^2)}; \quad L = \left(\frac{\ln M/\Lambda}{\ln M/\mu}\right)^{4/9}; \qquad (11)$$

$$\Lambda = 0.5\ \mathrm{GeV}, \mu = 0.15\ \mathrm{GeV},$$

where the first two orders of the operator expansion are taken into account (Figs. 1 and 2).

In Eq. (10) the left side, disregarding the quark masses, does not depend on the fact that the medium consists of nucleons; in this case we have $\langle M|\bar{u}\gamma_0 u|M\rangle = \rho n_u$, where n_u is the number of u quarks in the nucleon, and $\langle M| - (\alpha_s/\pi)G^2|M\rangle = 8m\rho/9$. In Eq. (11), $\langle M|\bar{d}d|M\rangle = \rho\langle N|\bar{d}d|N\rangle$ for the free nucleons. In the case of bound nucleons it should be taken into account that they, having exchanged pions (in the chiral limit only the single-pion exchange survives),[4] cannot acquire a momentum lower than the Fermi momentum. It follows that

$$\langle M|\bar{q}q|M\rangle = \rho\langle N|\bar{q}q|N\rangle - \frac{9}{2\pi}\rho^{4/3}\langle\pi|\bar{q}q|\pi\rangle\frac{1}{m_\pi^2}, \qquad (12)$$

$$\langle N|\bar{q}q|N\rangle = \frac{2\sigma}{m_u + m_d} = \kappa; \quad \langle\pi|\bar{q}q|\pi\rangle = \frac{m_\pi^2}{m_u + m_d},$$

where $m_{\pi,(u,d)}$ is the mass of a pion (u, d quark).

Taking into account the motion of nucleons in matter and using Eq. (7), we obtain, after minimizing the difference between the left side and the right side of Eqs. (10) and (11), the expression

$$U = [(-34 - 9{,}4\kappa)\zeta + 54\zeta^{4/3} + (3 + 0{,}4\kappa)\zeta^{5/3}]\ \text{MeV}, \qquad (13)$$

where $\zeta = \rho/\rho_{ph}$, and $\rho_{ph} = 0.17\ \text{fm}^{-3}$ is the phenomenological value of the equilibrium density. The equilibrium density ρ thus depends on the exact value of the σ term. At $\sigma = 50$ MeV (Refs. 5 and 6) for the equilibrium density ρ and the binding energy ϵ we find

$$\rho/\rho_{ph} = 2{,}25; \quad \epsilon = -16\ \text{MeV}; \qquad (14)$$

i.e., the description of the material is qualitatively correct.

Higher-order terms of the expansion of $U(\rho)$ can be incorporated by introducing into Eq. (11) some unknown condensates which are determined from the sum rules. As a result, we obtain

$$U = [(-34 - 9{,}4\kappa)\zeta + 54\zeta^{4/3} + (3 + 0.4\kappa)\zeta^{5/3} + (2{,}3\kappa - 1)\zeta^2 + (-0{,}2\kappa - 5)\zeta^{7/3}$$

$$+ (3 - 0{,}1\kappa)\zeta^{8/3}]\ \text{MeV}, \qquad (15)$$

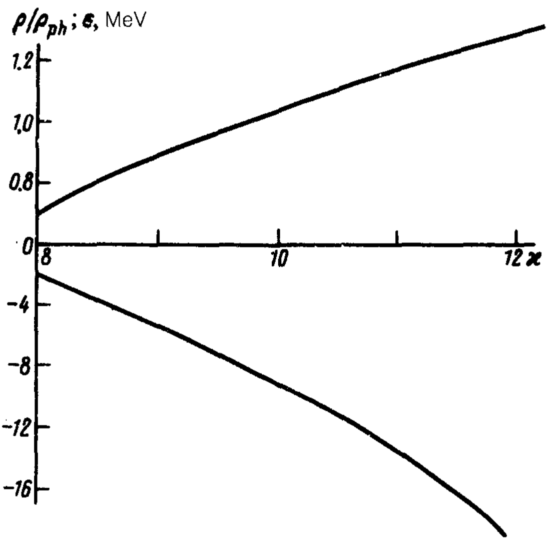

FIG. 3. Equilibrium density ρ (in units of $\rho_{ph} = 0.17\ \text{fm}^{-3}$) and binding energy versus κ.

which gives us the following values for $\sigma = 60$ MeV (Ref. 6):

$$\rho = 0.197 \text{ fm}^{-3}; \ \epsilon = -13 \text{ MeV}; \ \Delta\lambda^2 = 0.23 \text{ GeV}^6. \tag{16}$$

The plot of ρ and ϵ versus κ is shown in Fig. 3. This corresponds, from (16), to a distention of a nucleon by $\sim 3\%$ in matter.

In summary, we have determined the basic properties of nuclear matter on the basis of the QCD sum rules. This approach appears to us to be promising, since it allows the phenomena occurring at large spatial separations in nuclei to be linked with the hard processes occurring in them.

We wish to thank E. A. Akhmedov, Ya. Ya. Balitskiĭ, B. L. Birbrauer, V. M. Braun, B. L. Ioffe, A. V. Kolesnichenko, M. G. Ryskin, E. A. Sapershteĭn, V. A. Khodel', and M. A. Shifman for useful discussions.

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Translated by S. J. Amoretty