Gap splitting in a two-dimensional superconductor without an inversion center

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The superconducting state of two-dimensional electrons in a system in which there is no reflection symmetry in the plane of motion is studied. The loss of the inversion center may result in the appearance in the spectrum of Cooper pairs of the two energy gaps.

Some substances which exhibit superconducting properties have a layered electronic structrue. If such a substance has a complex composition and a large unit cell, many other nuclei and ions can be found between the atomic planes responsible for the conductivity. The ions surrounding the conducting layer are generally arranged not

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necessarily symmetrially with respect to the plane of the layer. Let us assume that tunneling coupling between the layers is weak. Let us examine one such layer. The loss of symmetry in the nearest neighborhood causes the two normals to be nonequivalent with respect to the layer, causing the appearance of a spin-orbit term in the electron Hamiltonian

$$\mathcal{H}_{so} = \alpha[\mathbf{pc}]\vec{\sigma}, \tag{1}$$

where $\mathbf{c}$ is a unit vector of one of the nonequivalent normals, and $\vec{\sigma}$ are the Pauli matrices. This situation was discussed previously in application to electronic layers in semiconductor heterojunctions.[1,2] We assume that the parameter $\delta = \alpha m/p_0$ is small, where $p_0 = (2m\mu)^{1/2}$, and μ is a chemical potential. We also assume that there is an s-type pairing interaction

$$\mathcal{H}_{pair} = \frac{\lambda}{2} \Sigma g_{\alpha\beta} a^+_{\mathbf{p}\alpha} a^+_{-\mathbf{p}\beta} u(p) u(q) a_{-\mathbf{q}\gamma} a_{\mathbf{q}\rho} g_{\rho\gamma}, \tag{2}$$

where $\hat{g} = i\sigma_2$, and the functions $u(p)$ are normalized by the condition $u(p_0) = 1$. The factorized form of the interaction was chosen only to simplify the calculations. In the absence of H_{s0} the particle spectrum is assumed for the same reason to be isotropic, $\epsilon_0(p) = p^2/2m$. In the normal state the energy surface has two branches

$$\epsilon_{(\pm)}(p) = \epsilon_0(p) \pm \alpha p \tag{3}$$

and the Fermi surface consists of two circles with the radii $p_{(\pm)} \approx p_0 \mp \alpha m$. For the states of the branch $\epsilon_{(+)}$ the spin-quantization axis is directed along $\mathbf{p}\times\mathbf{c}$, so the particle pair with the opposite momenta also has oppositely directed spins. For the states of the $\epsilon_{(-)}$ branch the quantization directions are opposite. All the states of the $\epsilon_{(+)}$ branch thus have a positive helicity which is opposite to that of the states of the $\epsilon_{(-)}$ branch.

At $T < T_c$ the equations of motion of the Green's functions are standard

$$[i\epsilon - \hat{H}_0(\mathbf{k})]\, G(\mathbf{k}, i\epsilon) = \hat{1} + \hat{\Delta}(k)\hat{F}^+(\mathbf{k}, -i\epsilon),$$
$$[i\epsilon - \hat{H}_0^t(-\mathbf{k})]\hat{F}^+(\mathbf{k}, -i\epsilon) = \hat{\Delta}^+(k)\hat{G}(\mathbf{k}, i\epsilon),$$

where (4)

$$\hat{H}_0(\mathbf{k}) = k^2/2m + \alpha[\mathbf{kc}]\vec{\sigma}, \quad \hat{\Delta}(k) = \hat{g}\Delta(k),$$
$$\Delta(k) = -\lambda \int \frac{d^2q}{(2\pi)^2} u(k) u(q) T \sum_{\epsilon} \mathrm{Tr}\, \hat{g}\hat{F}(\mathbf{q}, i\epsilon).$$

The superscript t denotes transposition and the evaluation of all Green's functions is the same as that adopted by Rainer and Serene.[3] The solutions of these equations are

$$\hat{G}(\mathbf{p},i\epsilon)=\hat{\Pi}^{(+)}(\mathbf{p})G_{(+)}(p,i\epsilon)+\hat{\Pi}^{(-)}(\mathbf{p})G_{(-)}(p,i\epsilon),$$
$$\hat{F}^{+}(\mathbf{p},-i\epsilon)=\hat{g}^{t}[\hat{\Pi}^{(+)}(\mathbf{p})F^{+}_{(+)}(p,-i\epsilon)+\hat{\Pi}^{(-)}(\mathbf{p})F^{+}_{(-)}(p,-i\epsilon), \tag{5}$$
$$F^{+}_{(\pm)}=-\overset{*}{\Delta}(p)/\epsilon^2+\xi^2_{(\pm)}+|\Delta(p)|^2,\quad G_{(\pm)}=-(i\epsilon+\xi_{(\pm)})/\epsilon^2+\xi^2_{(\pm)}+|\Delta(p)|^2,$$
$$\xi_{(\pm)}=\epsilon_{(\pm)}-\mu,\quad \Pi^{(\pm)}(\mathbf{p})=(1\pm[\hat{\mathbf{p}}\mathbf{c}]\vec{\sigma})/2.$$

Assuming $\Delta(p)=u(p)\Delta_0(T)$, we find the self-consistency condition

$$1=-\lambda\int\frac{d^2p}{(2\pi)^2}T\sum_{\epsilon}\sum_{\nu=\pm}u^2(p)/\epsilon^2+\xi^2_{(\nu)}+u^2(p)\Delta_0^2\,. \tag{6}$$

The momentum integral is concentrated in the region $(p-p_0)v_0\sim\omega_D$, where $v_0=p_0/m$, ω_D is the cutoff frequency. Assuming $u(p)\approx 1+\beta(p-p_0)/p_0$ and $\beta\sim 1$, we see that the right side of (6) is an even function of α and that it does not contain any linear corrections in the parameter δ. It may therefore be assumed that in it $\alpha=0$. Within an error of δ^2 the order parameter is therefore $\Delta_0(T)=\Delta_{BCS}(T)$. The difference in the energy gaps in the two Fermi circles

$$\Delta(p_{(-)})-\Delta(p_{(+)})\approx 2\beta\delta\Delta_0(T) \tag{7}$$

is of the first order in δ. Accordingly, the excitation energies of the particles with various helicities are not the same: A dynamic symmetry breaking occurs.

The splitting of the excitation spectrum in the superfluid Fermi systems was predicted even earlier (see, e.g., Ref. 4 and the bibliography cited there). In Ref. 4 and in our case the occurrence of this phenomenon is traceable in the final analysis to spin-orbit interaction. The difference stems from the fact that in Ref. 4 a system with a triplet vector pairing, at which the energy of the quasiparticle could depend on the projection of its spin onto the symmetry axis of the condensate, was analyzed. In the case considered here, however, the splitting is of a different geometric nature and is caused by the loss of spatial parity due to the singlet pairing, where the scalar condensate does not have a preferred spatial direction.

It should be emphasized that in contrast with Ref. 5, where bifurcation of the critical temperature was observed, splitting of the superconducting transition does not occur in the system we are considering. There is only one critical temperature and only one order parameter. Different energies, however, are required to break a Cooper pair, which involves the formation of two quasiparticles with a positive or negative helicity. The foregoing arguments may prove applicable to a 2D defect and to a layered crystal without an inversion center in the crystal group. If the unit cell contains an even number of conducting planes, as is the case with most recently discovered high-T_c superconductors, this condition would no longer be a necessary condition.

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