Anapole moment of a neutrino in a dispersive medium

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The induced anapole moment of a neutrino in a medium with a spatial dispersion is calculated. While a neutrino in an isotropic medium does not have an induced anapole moment, in a ferromagnet this moment is nonzero and has a finite value.

1. The various interactions of Dirac and Majorana neutrinos with an electromagnetic field, which differ in *C, P, T*-symmetry properties, can be characterized by various electromagnetic multipole moments. A general type of multipole, and for Majorana neutrinos the only type, is the anapole which Zel'dovich introduced[1] as an electromagnetic characteristic for a *T*-invariant interaction **G**curl**B** which separately violates *P* and *C*. Here **B** is the magnetic field, and the pseudovector **G** is the intrinsic dipole moment of the fermion ($s = \frac{1}{2}$) in its proper frame of reference:

$$\mathbf{G} = eG^{vac}(0)\,\varphi^{+}\vec{\sigma}\varphi. \tag{1}$$

This moment is characterized by the magnitude of the anapole, $G^{\rm vac}(0)$, by the spin $\mathbf{s} = \varphi^{+}\vec{\sigma}\varphi$, and by the electric charge of the electron, e (here[2)] $\alpha = e^2 = 137^{-1}$; $\vec{\sigma}$ are the Pauli matrices; and φ is a spinor in the proper frame).

The corresponding part of the conserved electromagnetic field of any spinor particle ($q^{\mu}J_{\mu} = 0$), and for Majorana neutrinos the entire electromagnetic current in vacuum, is of the form

$$J_{\mu}(q) = \frac{eG^{vac}(q^2)}{2\sqrt{E_pE_{p'}}}\bar{u}(\mathbf{p}')[q^2\gamma_{\mu} - q_{\mu}\hat{q}]\gamma_5 u(\mathbf{p}) \ , \tag{2}$$

where $G^{\rm vac}(q^2)$ is the (anapole) form factor, which depends on the 4-momentum transfer $q_{\mu} = p'_{\mu} - p_{\mu}$; and $E_p = (m^2 + p^2)^{1/2}$.

It is obvious from the form of current (2) that there can be no electromagnetic radiation from an anapole in vacuum (and thus the name "anapole"), as was pointed out some time ago by Zel'dovich[3)] (Ref. 1). The only possibility is a local interaction of current (2) with an external current which is a source of an external electromagnetic field.

Calculating the form factor $G^{\rm vac}(q^2)$ in the single-loop approximation of the standard vacuum model of electroweak interactions is apparently a problem which has not previously been solved.

In concluding the introductory part of this letter, we wish to point out that current (2) is related to anapole moment (1). We will not use a multipole expansion of the current in a complete set of orthonormal solutions of the wave equation, as was done in Ref. 2. We recall that all the intrinsic multipole moments of an elementary particle are defined in its proper frame, which formally coincides with the Breit system, in which there is no energy component of the momentum transfer ($\omega = 0$ if $\mathbf{p} + \mathbf{p}' = 0$). By virtue of the latter property, we can introduce a three-dimensional distribution of the density of a point anapole moment: $\mathbf{g}(\mathbf{r}) = \int d^3k/(2\pi)^3 e^{i\mathbf{kr}}\mathbf{G}(-k^2)$. For an anapole which is at rest, the relationship between the density and (1) is

$$\mathbf{g}(\mathbf{r}) = \mathbf{G}\delta^3(\mathbf{r}), \text{ i.e.,} \qquad \mathbf{G} = \lim_{k \to 0} \mathbf{G}(-k^2).$$

The most general form of the coupling of the Fourier transform of $\mathbf{G}(-k^2)$ with the transverse 3-current

$$\mathbf{J}(\mathbf{k}) = \frac{eG^{vac}(-k^2)}{2E_p}\bar{u}(-\mathbf{p})[-k^2\vec{\gamma} + \mathbf{k}(\mathbf{k}\vec{\gamma})]u(\mathbf{p})$$

[see (2)] is thus[4)]

$$G_m(-k^2) = a\frac{\partial^2 J_n(\mathbf{k})}{\partial k_m \partial k_n} + b\frac{\partial^2 J_m(\mathbf{k})}{\partial \mathbf{k}^2}. \qquad (3)$$

There are two requirements which make it possible to determine the coefficients a and b in (3). First, the quantity in (3) must be equal to (1), i.e., we must have $G_m = (e/2m_\nu)G^{\text{vac}}(0)\bar{u}(0)\gamma_m\gamma_5 u(0)$, in the proper frame ($\mathbf{k} = -2\mathbf{p} = 0$). This condition yields the equation $2a - 4b = 1$. Second, the longitudinal current $\mathbf{J}_{\|} = \nabla\varphi$—a source of electric multipoles—must not contribute to the anapole moment $\mathbf{G} = \int d^3r\mathbf{g}(\mathbf{r})$ with a density $g_m(\mathbf{r}) = -[ar_m(\mathbf{rJ}(\mathbf{r})) + br^2 J_m(\mathbf{r})]$, which corresponds to (3). From this requirement we find the equation $2a + b = 0$.

The coefficients which we thus find, $a = 1/10$ and $b = -1/5$, correspond to a so-called toroidal dipole moment with a density $-\mathbf{g}(\mathbf{r}) = [(\mathbf{Jr})\mathbf{r} - 2r^2\mathbf{J}(\mathbf{r})]/10$ (Ref. 2). From (3) we finally find a general expression for the anapole moment in a medium and in vacuum:

$$G_m = \frac{e}{20m_\nu}\bar{u}(0)\left[\frac{\partial^2\Gamma_n(0,\mathbf{k})}{\partial k_m \partial k_n} - 2\frac{\partial^2\Gamma_m(0,\mathbf{k})}{\partial \mathbf{k}^2}\right]u(0)\Big|_{\mathbf{k}=0}, \qquad (3')$$

where, in the case of a medium which is at rest as a whole, the arguments of the electromagnetic vertex $\Gamma_\mu(\omega,k)$ are chosen to suit the frequency spectrum of the external field which is acting on the medium. Expression (3') is valid for a static field; it is assumed that the particle (the neutrino) is at rest (is stopped in the medium).

2. Substitution of the electromagnetic vertex of a neutrino in an isotropic medium[3] into expression (3') yields a vanishing result: $G_m = 0$. In a magnetically ordered medium, e.g., a ferromagnet, the static electromagnetic vertex of a Dirac neutrino is[4]

$$\Gamma_n(0,\mathbf{k}) = \frac{G_V(1+\gamma_5)\gamma_i}{8\pi\alpha\sqrt{2}} e_{imp}\, e_{kln}\, k_m k_l (\mu^{-1}_{pk}(0,\mathbf{k}) - \delta_{pk})$$

$$+ \frac{G_F m_e (1+\gamma_5)\gamma_i}{4\pi\alpha\sqrt{2}} e_{nkp}\, k_p (\mu^{-1}_{ik}(0,\mathbf{k}) - \delta_{ik}), \tag{4}$$

where $G_F = 10^{-5}/m_p^2$ is the Fermi weak-interaction constant, m_p is the mass of a proton, $\mu_{ik}(\omega,\mathbf{k})$ is the magnetic permeability tensor of the ferromagnet, $G_V = G_F(1+4\xi)$, $\xi = \sin^2\theta_w$, and θ_w is the Weinberg angle.

The second term in (4) [which we denote by $\vec{\Gamma}_2(0,\mathbf{k})$] originates from the statistical averaging of the pseudovector electron current, $\langle\bar{\psi}_e\gamma_\mu\gamma_5\psi_e\rangle$, and determines the induced magnetic moment of a neutrino in a ferromagnet.[4] The corresponding 3-current $\mathbf{J}_2(\mathbf{r}) = \int d^3k/(2\pi)^3 \times e^{ikr}\bar{u}(0)\vec{\Gamma}_2(0,\mathbf{k})u(0)/2m_\nu$, is $\mathbf{J}_2(\mathbf{r}) = \mathrm{curl}\vec{\mu}(\mathbf{r})$, i.e., a circular current (a "sheet"), which generates a magnetic moment $\vec{\mu} = \int d^3r\vec{\mu}(\mathbf{r}) = -2\sqrt{2}G_F m_e^2\mu_B\varphi^+\vec{\sigma}\varphi/4\pi\alpha$ (cf. Ref. 3). The vertex $\vec{\Gamma}_2(0,\mathbf{k})$ does not contribute to anapole moment (3′) because the tensor $\mu_{ik}(0,k)[=\delta_{ik}\mu^{-1}(0,k)]$ is an even function of the wave vector k.

The first term in (4) [which we denote by $\vec{\Gamma}_1(0,\mathbf{k})$] stems from the statistical averaging of the vector current, $\langle\bar{\psi}_e\gamma_\mu\psi_e\rangle$, and under the simplest assumption, that the tensor $\mu^{-1}_{ik} = \mu^{-1}\delta_{ik}$ is diagonal, it leads to the following density of the anapole moment induced in the ferromagnet:

$$\mathbf{g}(\mathbf{r}) = \frac{eG_V\varphi^+\vec{\sigma}\varphi}{8\pi\alpha\sqrt{2}} \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{k}\mathbf{r}}(1-\mu^{-1}(0,k)), \tag{5}$$

which is coupled with the poloidal current density $\mathbf{J}_1(\mathbf{r}) = \int d^3k/(2\pi)^3 e^{i\mathbf{kr}}\bar{u}(0)\vec{\Gamma}_1(0,\mathbf{k})u(0)/2m_\nu$ by the relation $J_1(\mathbf{r}) = \mathrm{curlcurl}\mathbf{g}(\mathbf{r})$. The current forms loops which are wound around a torus with an axis along the spin of the neutrino.[5)]

The magnitude of the induced anapole (toroidal) moment, $\mathbf{G} = \int \mathbf{g}(\mathbf{r})d^3r = eG^{ind}_{med}\varphi^+\vec{\sigma}\varphi$, is

$$G^{ind}_{med} = \frac{G_V(\mu(0,0)-1)}{8\pi\alpha\sqrt{2}\mu(0,0)}. \tag{6}$$

In a ferromagnet $[\mu = \mu(0,0) \gg 1]$ it has a nonzero value.

[2)] We are using a system of units with $\hbar = c = 1$; the Feynman metric $q^2 = q_\mu q^\mu = \omega^2 - \mathbf{k}^2, \mu = 0,1,2,3$; and the standard representation of the Dirac γ matrices. Here $\gamma_5 = \gamma_5^+ = i\gamma^0\gamma^1\gamma^2\gamma^3$, and the Latin indices take on the values $i,m,n = 1,2,3$.

[3)] In addition to the Lorentz condition $A_\mu/x_\mu = 0$ for vacuum photons the condition of gauge invariance of the second kind holds. In other words, the spectrum of these photons is massless: $q^2 = 0(\Box A_\mu = 0)$. In a medium, with $q^2 \neq 0$, radiation is possible in principle.

[4)] In view of the dependence of the vertex on $\mathbf{k} = -2\mathbf{p}$, the argument $\mathbf{k}$ can be omitted from the bispinors $u(p)$, from the normalization factor $(E_p \to m_\nu)$, and from the form factor $G^{\mathrm{vac}}(-k^2)$. In this case, the quantity in (3) is no longer dependent on k. This property corresponds to a point anapole.

[5)] More precisely, the torus is made up of closely packed circular currents.

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