

Active suppression of dephasing in Josephson-junction qubits

D. V. Averin, R. Fazio⁺

Department of Physics and Astronomy, SUNY Stony Brook, Stony Brook, NY 11794-3800

+NEST-INFM & Scuola Normale Superiore, 56126 Pisa, Italy

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Simple majority code correcting k dephasing errors by encoding a qubit of information into $2k + 1$ physical qubits is studied quantitatively. We derive an equation for quasicontinuous evolution of the density matrix of encoded quantum information under the error correction procedure in the presence of correlated dephasing noise. A specific design of a Josephson-junction nanocircuit implementing this scheme is suggested.

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Josephson qubits are among the most promising devices to implement solid state quantum computation [1, 2]. Quantum manipulations of individual [3–9] and coupled [10] qubits has been demonstrated experimentally. At present, an important task is to develop strategies to minimize the effects of decoherence on the dynamics of the qubit systems. Error-avoiding schemes suggested for reduction of decoherence either employ symmetries of the qubit-environment interaction to create areas of the Hilbert space not affected by decoherence [11, 12] or use rapid random dynamic perturbations of the system to average out the effects of external noise [13, 14]. Both approaches appear to be problematic for the solid-state qubits where noise typically does not have any particular symmetry and its correlation time is short. This leaves error-correction as the main strategy for suppression of decoherence in solid-state qubits. In this work, we suggest an implementation of one of the basic error-correction algorithms for the suppression of dephasing errors (which can be expected to be the dominant type of errors in solid-state circuits – see, e.g., Ref. [9]), and develop its quantitative description. Our implementation employs Josephson-junction qubits that combine charge and flux dynamics [15, 16, 6], and requires only a small number of qubit transformations to operate.

An interesting feature of the scheme considered in this work is the possibility of developing its detailed quantitative description within the realistic model of the qubit-environment interaction. Most importantly we have analyzed the effect of the correlations in the noise acting on different qubits. While discussions of the error-correction rely typically in an essential way on independent noise models, environments of the solid-state qubits can be to a large degree correlated because of the finite distance between qubits in a circuit. A

clear illustration of this is provided by the background charge fluctuations that are the main source of dephasing in charge qubits [3, 6, 10, 17]. Long-range nature of the Coulomb interaction creates noise correlations by coupling the qubits to the same charge fluctuators.

We consider specifically the problem of a “quantum memory”. The task here is to preserve the stationary state of the qubit in the presence of dephasing noise. The Hamiltonian of the qubit register contains then only the coupling to the environment. Within the standard oscillator model of the reservoir [18–21] this coupling is:

$$H = \sum_j \sigma_z^{(j)} \xi_j, \quad (1)$$

$$\xi_j = \sum_{m,\omega} [\lambda_{m,j}(\omega) a_{m,\omega} + \lambda_{m,j}^*(\omega) a_{m,\omega}^\dagger].$$

Here we assumed several independent ensembles of environmental oscillators (numbered by m), as needed to model different profiles of spatial correlations of random forces ξ_j . The index $j = 1, \dots, N$ in (6) labels the qubits, and coefficients $\lambda_{m,j}(\omega)$ are coupling constants of the qubit j to the oscillators of reservoir m in the mode ω and creation/annihilation operators $a_{m,\omega}, a_{m,\omega}^\dagger$. Time evolution of the “qubits+environment” system is described conveniently in the interaction representation, and the evolution operator $U(t)$ is:

$$U(t) = \exp \left\{ -i \sum_j \varphi_j(t) \sigma_z^{(j)} \right\} U_r(t), \quad (2)$$

$$U_r(t) = \exp \left\{ i \sum_{m,\omega} \frac{\omega t - \sin \omega t}{\omega^2} \left| \sum_j \lambda_{m,j} \sigma_z^{(j)} \right|^2 \right\}.$$

The first term in $U(t)$ represents fluctuating phases $\varphi_j(t)$ of the qubit basis states induced by the environmental forces $\xi_j(t)$: $\varphi_j(t) = \int_0^t \xi_j(t') dt'$. The second term,

$U_r(t)$, results from the renormalization of the qubit parameters by the qubit-environment interaction. To see this more explicitly, we note that the sum over frequencies ω in this exponent has a natural cut-off at large frequencies $\omega \simeq \tau_c^{-1}$, where τ_c is the time scale at which environment forces acting on different qubits are correlated. For weak decoherence we are interested in the time scales much larger than τ_c . In this regime, the phase represented by $U_r(t)$ is dominated by the term that grows linearly with t , and can be viewed as arising from the renormalization of the qubit energy. Equation (7) shows that such a renormalization includes a shift of the total energy of the register and a term which induces an effective qubit-qubit interaction. The total energy shift is irrelevant as long as we consider an individual register. Neglecting it, we see that $U_r(t)$ results from the Hamiltonian evolution with the Hamiltonian

$$H_r = - \sum_{j,j'} V_{jj'} \sigma_z^{(j)} \sigma_z^{(j')}, \quad V_{jj'} = 2\text{Re} \sum_{m,\omega} (\lambda_{m,j}^{(\omega)} \lambda_{m,j'}^{(\omega)*} / \omega). \quad (3)$$

In dropping the oscillatory term one should be careful for the convergence of the sum over frequencies. In the case of an Ohmic bath, for example, this can be done and leads to an effective interaction which scales with the dissipation strength and with the cut-off of the environmental modes. The qubit-qubit interaction strength $V_{jj'}$ is non-vanishing only if the same reservoir m couples to more than one qubit, so that the reservoir forces ξ_j at different qubits are correlated. The time evolution with the Hamiltonian H_r , and more generally, the evolution operator U_r in Eq. (7) represent the deterministic part of the qubit evolution induced by the qubit-reservoir interaction. Nevertheless, it is more appropriate to still treat it as dephasing, because $V_{jj'}$ are unknown and in general incommensurate quantities.

The time evolution of the density matrix $\rho(t)$ of the qubit register is obtained from Eq. (7) through the relation $\rho(t) = \text{Tr}_{env} \{ U^\dagger(t) \sigma(0) U(t) \}$, where σ is the total density matrix of the “qubits+environment” system. The environment will dephase the qubits if they are prepared initially in the state $\rho(0)$ that is uncorrelated with the state of the environment, $\sigma(0) = \rho_{env} \rho(0)$. Assuming that the environment is in thermal equilibrium at temperature Θ , and no error correction procedure is applied, we get using the standard property of the Gaussian noise:

$$\rho(t) = \exp \left\{ -\frac{1}{2} \sum_{j,j'} \langle \varphi_j(t) \varphi_{j'}(t) \rangle (\sigma_z^{(j)} - \bar{\sigma}_z^{(j)}) \times \right. \\ \left. \times (\sigma_z^{(j')} - \bar{\sigma}_z^{(j')}) \right\} \cdot U_r^\dagger(t) \rho(0) U_r(t). \quad (4)$$

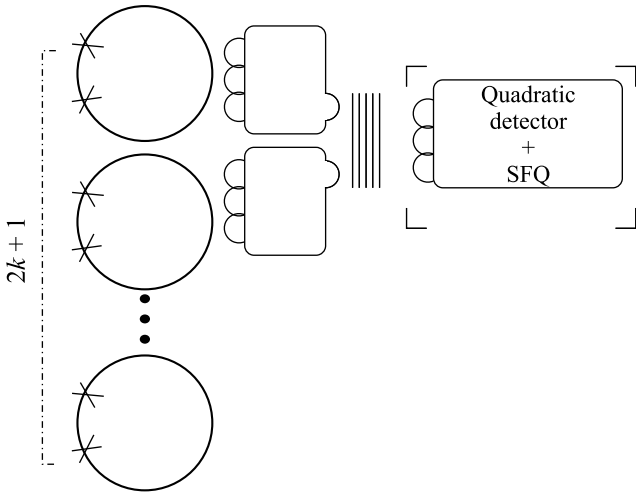
Here we introduced the convention that the bar over σ_z operators means that they act on ρ from the right. Qualitatively, Eq. (4) shows that the matrix elements of ρ that are further away from the diagonal in the σ_z basis decay faster. The diagonal elements (on which $\sigma_z - \bar{\sigma}_z = 0$) remain constant. In the case of one physical qubit, Eq. (4) gives $\rho(t) = e^{-\langle \varphi^2(t) \rangle (1 - \sigma_z \bar{\sigma}_z)} \rho(0)$, i.e., the off-diagonal elements of ρ are suppressed with time as $e^{-2\langle \varphi^2(t) \rangle} \equiv e^{-P(t)}$. If the environment density of states is Ohmic, i.e., $\sum_{m,\omega} |\lambda_m(\omega)|^2 \dots = g \int_0^\infty d\omega \omega e^{-\omega \tau_0} \dots$, direct evaluation for $\Theta \ll 1/\tau_0$ gives: $P(t) = 2g \ln[\sinh(\pi t \Theta) / \pi \tau_0 \Theta]$. At large t , when the random force ξ appears δ -correlated, $P(t)$ reduces to $P(t) = \Gamma t$, where $\Gamma = 2\pi g \Theta$ is the dephasing rate.

One can reduce the effective dephasing rate, by the encoding that corrects the phase errors [22, 23]. Generalized to k errors, this encoding is:

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha \underbrace{|+\dots+\rangle}_{2k+1} + \beta \underbrace{|-\dots-\rangle}_{2k+1}. \quad (5)$$

In Eq. (5), a bit of quantum information is encoded in the state of the $2k+1$ physical qubits, and the $|\pm\rangle$ states of each of these qubits are obtained through the Hadamard transform \hat{H} (the $\pi/2$ -rotation around y axis) from the $|0,1\rangle$ states. All of the σ_z operators in the time evolution, Eq. (7), are changed by \hat{H} : $\hat{H} \sigma_z \hat{H} = \sigma_x$, so that for the states on the right-hand-side of Eq. (5) the dephasing looks like transitions between the $|\pm\rangle$ states of each qubit, and can be directly detected by measurements in this basis and corrected by applying simple pulses returning the qubit into the initial state. The error-detecting measurements, however, should not destroy the quantum information encoded in the state (5), i.e., it should not distinguish the α and β parts of this state. This condition is not satisfied by measurements on individual qubits but can be satisfied by measurements on pairs of the nearest-neighbor qubits comparing their states. Despite the apparent complexity of this scheme, it has quite natural implementation in the Josephson-junction qubits – see Figure.

To describe this process quantitatively we assume that its measurement/correction part can be done on the time scale that is much shorter than the one set by characteristic dephasing rate Γ . Different terms in the environment-induced evolution of the encoded state, Eq. (5), during the time interval T between the successive application of the “measurement+correction” operations can be conveniently classified by the number of qubits flipped during this time interval. In the relevant regime of sufficiently short T : $P(T) \ll 1$, the probability amplitude of these terms decreases rapidly when this number increases. If we keep only the terms that flip up



Schematics of the Josephson-junction circuits implementing suppression of dephasing. Crosses denote tunnel junctions, the electrodes between them act as charge qubits. Monitored currents in the nearest-neighbor loops enclosing qubits allow to detect dephasing errors

to k qubits, we see directly from Eq. (7) that the time evolution at this level of accuracy (denoted by $U_k(T)$) preserves the superposition of the α and β parts of the encoded state:

$$U_k(T)[\alpha|\oplus\rangle + \beta|\ominus\rangle] = \sum_q [\alpha|\psi_q\rangle + \beta\hat{R}|\psi_q\rangle]u_q. \quad (6)$$

Here index q runs over 2^{2k} different register states obtained from the state $|\oplus\rangle \equiv |+\dots+\rangle$ by flipping up to k qubits, u_q are the probability amplitudes of these states, $\hat{R}|\psi_q\rangle$ denotes the state $|\psi_q\rangle$ with all $2k+1$ qubits inverted, and $|\ominus\rangle \equiv |-\dots-\rangle$.

The measurements that compare the qubit states in all pairs of the nearest-neighbor qubits do not distinguish states $|\psi_q\rangle$ and $\hat{R}|\psi_q\rangle$, and therefore also preserve the superposition of the α and β terms in Eq. (6). The 2^{2k} different outcomes (“equal” or “different”) of the $2k$ such measurements distinguish all terms with different q in Eq. (6) and enable one to decide what qubits were flipped during the time interval T . Application of the correcting pulses should then bring the state of the qubit register back to its initial form, Eq. (5), so that the encoded quantum state does not change in this approximation. The residual evolution of the encoded state is associated with the possibility that the environment flips more than k different qubits; for $P(T) \ll 1$ – precisely $k+1$ qubits. Following the same steps as above, we see that when $k+1$ qubits are flipped, the measurement/correction cycle interchanges the α and β weights in the encoded state (5), with probability p . Since p is small, $p \ll 1$, the encoded state changes substantially

only on the time scale larger than the period T of one error-correction cycle, and its evolution on this scale can be conveniently described by the continuous equation for the density matrix $\rho^{(c)}$ in the basis of $|\oplus\rangle$ and $|\ominus\rangle$ states:

$$\dot{\rho}_{++}^{(c)} = \frac{\gamma_k}{2}(\rho_{--}^{(c)} - \rho_{++}^{(c)}), \quad \dot{\rho}_{+-}^{(c)} = \frac{\gamma_k}{2}(\rho_{-+}^{(c)} - \rho_{+-}^{(c)}). \quad (7)$$

Here $\gamma_k \equiv 2p/T$ is the effective dephasing rate of the encoded quantum information, the superscript (c) indicates that $\rho^{(c)}$ is the reduced density matrix in the presence of error correction. Thus, our error-correcting procedure replaces the dephasing in the individual physical qubit with the dephasing of encoded quantum information at a smaller rate. Indeed, if one writes Eqs. (7) in the rotated basis $|\oplus\rangle \pm |\ominus\rangle$, they explicitly acquire the form characteristic for pure dephasing: constant diagonal elements of the density matrix and decay of the off-diagonal elements with the rate γ_k . The dephasing rate γ_k can be calculated from the evolution operator (7).

For $k=1$, when the relevant errors flip 2 out of 3 qubits, we get:

$$\gamma_1 = \frac{2}{T} \sum_{j>j'} (T^2 V_{jj'}^2 + 2\langle\varphi_j(T)\varphi_{j'}(T)\rangle^2 + \langle\varphi_j^2(T)\rangle\langle\varphi_{j'}^2(T)\rangle).$$

The first two terms in this expression represent the contribution to dephasing from noise correlations at different qubits, while the last term exists also for uncorrelated noise. If the noise is δ -correlated in time, γ_1 reduces to $\gamma_1 = T \sum_{j>j'} (2V_{jj'}^2 + \Gamma_{jj'}^2 + \Gamma_j\Gamma_{j'}/2)$, where Γ_j is the dephasing rate in the j th qubit, and $\Gamma_{jj'}$ is introduced through $2\langle\varphi_j(t)\varphi_{j'}(t)\rangle = \Gamma_{jj'}t$.

If the dephasing forces at different qubits are uncorrelated, the encoded dephasing rate can be easily calculated for arbitrary k :

$$\gamma_k = \frac{1}{2^k T} \sum_{j_1>j_2>\dots>j_{k+1}} P_{j_1}(T)P_{j_2}(T)\dots P_{j_{k+1}}(T), \quad (8)$$

and one sees that γ_k decreases exponentially with the “degree of encoding” k . When the probabilities $P(T)$ of dephasing errors in individual qubits can be expressed through the dephasing rate Γ , Eq. (8) reduces to $\gamma_k = \Gamma(\Gamma T)^k (2k+1)!/(2^k k!(k+1)!)$, if Γ is the same for all qubits.

Exponential suppression of γ_k with k is limited in the scheme considered above by possible imperfections of the measurement/correction operations. The most important is direct dephasing of the encoded states by measurements, which, in contrast to correction steps, need to be performed each period T . For example, one

of the specific non-idealities of measurement detectors that leads to direct dephasing of the state is residual linear response coefficient of the quadratic detectors needed to perform pair-wise comparison of the qubit states – see Eq.(10) below. Linear terms couple the detector directly to the $|\pm\rangle$ states of individual qubits and introduce finite phase shifts between them. Since the number of required measurements is proportional to k , the rate of introduced dephasing should also be proportional to k , and can be denoted as $\bar{\gamma}k$. The effect of this dephasing on the evolution of encoded quantum information is described then by adding the usual dephasing term to the equation for the off-diagonal element of the density matrix $\rho^{(c)}$ (7):

$$\dot{\rho}_{+-}^{(c)} = \frac{\gamma k}{2}(\rho_{-+}^{(c)} - \rho_{+-}^{(c)}) - \bar{\gamma}k\rho_{+-}^{(c)}. \quad (9)$$

Qualitatively, the two types of dephasing processes in Eq. (9) have similar effect of suppressing the fidelity of the encoded state, but depend differently on k . The optimum degree of encoding is estimated crudely by minimizing the total dephasing rate: $k_{opt} \sim \ln(\bar{\gamma}/\Gamma)/\ln(T\Gamma)$. One obvious result of this optimization is that for the considered scheme of the dephasing suppression to make sense, the dephasing introduced by imperfections of the correcting procedure should be much weaker than the original qubit dephasing Γ .

This condition can be satisfied in Josephson-junction qubits, where the dynamics of magnetic flux characterized by longer coherence times (at least tens of nanoseconds – see, e.g., [9]) can be used to suppress dephasing in charge-based qubits. Charge qubits have quite short decoherence times, ~ 1 ns [3, 10], limited by the background charge fluctuations, but offer some advantages, e.g., demonstrated simplicity of the qubit-qubit coupling [10]. Therefore it would be of interest to use the approach discussed in this work to suppress the dephasing of charge degrees of freedom with the help of controlled flux dynamics. A sketch of the possible setup achieving this is shown in Figure. Its main elements are the charge qubits, formed by two small tunnel junctions in series, enclosed in small superconducting loops threaded by magnetic flux Φ equal to half of the magnetic flux quantum Φ_0 . It can be shown [16] that the current in each loop represents the σ_x component of the qubit dynamics, and its monitoring measures therefore the qubit in the σ_x basis as needed for detection of the dephasing errors. Comparison of the states of the nearest-neighbor qubits can be achieved by measuring not directly the currents in the loops, but the square of the difference

(or of the sum) between the currents. Such a quadratic detection measures the product operator $\sigma_x^{(j)}\sigma_x^{(j+1)}$:

$$(\sigma_x^{(j)} \pm \sigma_x^{(j+1)})^2 = 2(1 \pm \sigma_x^{(j)}\sigma_x^{(j+1)}), \quad (10)$$

and provides information on whether the states of the two qubits are the same or not without measuring them. Quadratic measurements can be realized by the usual magnetometers but operated at a point where the linear response coefficient vanishes. These measurements, subsequent classical calculations, and application of correction pulses, can be done with sufficient frequency by existing “SFQ” superconductor electronics compatible with the qubits [24].

In summary, we suggested a simple scheme of performing basic error-correction in Josephson-junction qubits. The scheme suppresses dephasing errors and can be analyzed quantitatively within the realistic model of environment, including the possibility of noise correlations at different qubits. If the errors introduced by the correction procedure are negligible, the residual dephasing rate for the encoded quantum information decreases exponentially with the degree of encoding.

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