$(\alpha')^2$ correction to the effective action of a boson string from the three-loop β function of a nonlinear σ model with torsion

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The three-loop β function of a nonlinear 2D σ model with a Wess-Zumino-Witten term satisfies the integrability condition. The results of a calculation of the $(\alpha')^2$ correction to the effective low-energy action of a boson string, with allowance for the antisymmetric Kalb-Ramon tensor field, are presented.

The σ-model approach in the string theory is one of the methods of analyzing the low-energy dynamics of the massless modes of the string in space-time. Calculation of the corresponding effective action is of crucial importance in the solution of problems of compactification and phenomenology of strings. The equations of motion which are deduced from the effective action are equivalent to the conditions for conformal invariance of the corresponding nonlinear 2D σ model.[1–2] The conformal invariance is closely linked with the vanishing of the perturbative β functions of the σ model.[3–4]

In the case of a boson string the two-loop $(0(\alpha')^2)$ β functions[1)] for the metric, for the antisymmetric tensor (the Kalb-Ramon field), and for the dilaton, which were calculated in Refs. 4–6, satisfy the integrability condition and the corresponding effective action is in agreement with the results obtained from the string amplitudes.[7]

The three-loop β function of a nonlinear 2D σ model with a Wess-Zumino-Witten term has been calculated in Refs. 8 and 9. In the supersymmetric case the three-loop contribution of the σ model with torsion to the β function vanishes.[10]

In the Bose case the three-loop contribution to the β function, with allowance for torsion, has an extremely complex form with the general structure

$$\beta^{(3)}_{df} \sim \hat{R} R^2 + (\hat{\nabla} R)^2 + R^2 \hat{\nabla} H + R(\hat{\nabla} R) H + (\hat{\nabla}^2 R)(\hat{\nabla} H) + \hat{R} R H^2 + R(\hat{\nabla} H)^2$$
$$+ (\hat{\nabla} R)(\hat{\nabla} H) H + RH(\hat{\nabla}^2 H) + (\hat{\nabla}^2 R) H^2 + RH^2 (\nabla H)^2 + (\hat{\nabla} R) H^3 + (\hat{\nabla} H)^3$$
$$+ (\hat{\nabla}^2 H)(\nabla H) H + (\hat{\nabla}^3 H) H^2 + \hat{R} H^4 + (\hat{\nabla} H)^2 H^2 + H^4 (\nabla H), \qquad (1)$$

where R is the curvature tensor (without torsion), H is the 3-form of the antisymmetric tensor strength, and $\hat{R}$ and $\hat{\nabla}$ are the curvature tensor and the covariant derivative, respectively, with a torsion of H (Ref. 6). The particular features of the calculations and the explicit form of the β function are given in Ref. 8.

One of the methods of restoring the effective action on the basis of the known β

function involves the use of the most general action with indefinite coefficients and the identity[5]

$$\nabla^a \beta^g_{ab} - H_b^{\ ac} \beta^H_{ac} = -\frac{1}{2}\nabla_b L_{eff}, \tag{2}$$

in which the effective Lagrangian $L_{\rm eff}$ is determined within an error of the total derivative.[2)]

In reconstructing the effective string action from the β function of the σ model it is necessary to take into account the dependence of the β function on the rules governing the use of the Levi-Civita symbol $\epsilon^{\mu\nu}$ (in terms of the dimensional regularization) and on the adopted subtraction scheme, and also the dependence of the action on the redefinition of the fields: redefinition of the metric and the antisymmetric tensor.[11] The use of the most general (in the given order in α') redefinition of the fields allows the introduction of a "contracted" action, whose definition contains a minimum number of terms. The result of such an analysis is the following action formula:

$$I_2 = -\frac{1}{32(2\pi)^2} \int d^{26}x\sqrt{g}[a_1 R_{abcd} R^{cd}{}_{pq} R^{pqab} + a_2 R_{abcd} R^{apc}{}_t R^{btd}{}_p$$

$$+ a_3 R_{abcd} H^{apq} H^b_{\ pt} H^c_{\ qs} H^{dts} + a_4 R_{abcd} H^{abp} H^{cdk} H_{ptq} H_k^{\ tq}$$

$$+ a_5 R_{abcd} H^{atk} H^{bsm} H_s^{\ cd} H_{tkm} + a_6 R_{abcd} H^{atk} H_t^{\ bs} H^{cdp} H_{ksp}$$

$$+ a_7 R_{abcd} H^{atk} H^b_{\ sm} H^c_{\ tk} H^{dsm} + a_8 R_{abcd} R^{abpq} H^{cdt} H_{pqt}$$

$$+ a_9 R_{abcd} R^{abpq} H^c_{\ pt} H_q^{\ dt} + a_{10} R_{abcd} R^{apcq} H^b_{\ pk} H_q^{\ dk}$$

$$+ a_{11} R_{abcd} R^{abct} H^{dpq} H_{tpq} + a_{12} R_{abcd} R^{apqt} H_p^{\ cd} H^b_{\ qt}$$

$$+ a_{13} H_{abc} H^{apq} H^{bct} H_{pqs} H_{tkm} H^{skm} + a_{14} H_{abc} H^{apq} H^{bct} H_{pts} H_{qkm} H^{skm}$$

$$+ a_{15} H_{abc} H^{apq} H^{bct} H_{psm} H_q^{\ sk} H^m_{\ tk} + a_{16} H_{abc} H^{apq} H^{bmt} H^{cks} H_{pmk} H_{qts}$$

$$+ a_{17} H_{abc} H^{apq} H^b_{\ pt} H^{cms} H_{qmk} H_s^{\ tk} + a_{18} H_{abc} H_k^{\ ap} D^b H^{cms} D^k H_{msp}$$

$$+ a_{19} H^2 (DH)^2 + a_{20} H^2 (DH)^2 + a_{21} R_{abcd} D^a H^b_{\ kt} D^c H^{dkt}], \tag{3}$$

where as the 19th and 20th terms we can use any two linearly independent expressions:

$$H_{abc}H^{apq}D_pH^b_{ms}D_qH^{cms}, \quad H_{abc}H^{ams}D_kH^b_{mt}D^kH^{ct}_s \quad ,$$

$$H_{abc}H^{pqt}D^aH^{bck}D_tH_{pqk}, \quad H_{abc}H^{pqt}D^aH_{pqk}D_tH^{bck} \quad . \tag{4}$$

Effective action (3) is consistent with the three-loop β function (in the Huff-Veltman prescription for $\epsilon^{\mu\nu}$) of a nonlinear 2D σ model with torsion[8] if the coefficients a_i in (3) are chosen in the following way:

$$\begin{aligned} &a_1=-2, \quad a_2=-8/3; \quad a_3=-2/3, \quad a_4=3/2, \quad a_5=9/2, \quad a_6=5/3, \\ &a_7=9/8; \quad a_8=7/4, \quad a_9=-4/3, \quad a_{10}=2/9, \quad a_{11}=-3, \quad a_{12}=2; \\ &a_{13}=880/81, \quad a_{14}=-281/27, \quad a_{15}=1045/81, \quad a_{16}=-2626/27, \\ &a_{17}=-2383/27; \quad a_{18}=3/2; \quad a_{19}=a_{20}=0; \quad a_{21}=3/2 \quad . \end{aligned} \tag{5}$$

The coefficients α_1, a_2, a_3, a_8, a_9, a_{10}, a_{11}, a_{12}, a_{16}, a_{17}, a_{18}, and a_{21} are invariant.

Equations (3) and (5) are the basic results. In the absence of torsion ($H=0$), these results are the same as the well-known Metsaev and Tseytlin's[14] results of the calculation of the gravitational part of the effective action from the tree amplitudes in the theory of closed boson strings.

The integrability of the three-loop β function of the nonlinear 2D σ model with the Wess-Zumino-Witten term is at the same time a control of the calculations of the β function and the effective action. At the same time, it would be highly desirable to calculate independently effective action (3) from the tree string amplitudes. This calculation would constitute the final test of the coefficients (5).

1) The agreement and notation used here are given in Ref. 6; in particular, $2\pi\alpha'=1$.
2) There are other ways in which the effective action can be restored; for example, by using CP identity.[12,13]

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