Nonlinear quantum conductance of a point contact

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An increase in the extraction voltage V is shown to round the quantum steps in a plot of the current through a point contact [B. J. van Wees *et al.*, Phys. Rev. Lett. **60**, 848 (1988); D. A. Wharam *et al.*, J. Phys. C. **21**, L209 (1988)] versus the diameter of the constriction. The number of steps on this plot becomes bounded: $n < 2\overline{E}_F/eV$. Additional steps, equal to half-integer values of the quantum $e^2/2\pi h$, appear in the differential conductance.

Recent experiments by van Wees *et al.*[1] and Wharam *et al.*[2] have revealed a quantization of the conductance G of a ballistic point contact in a two-dimensional electron system. Well-defined plateaus were observed in the plot og G versus the constriction diameter[1)] d at integer values of $(\pi\hbar/e^2)G$. A theory for this phenomenon[3] relates the appearance of the plateaus to an adiabatic passage of an electron wave across the point contact. The adiabatic nature of the passage stems from the smoothness of the constriction and implies that there can be an effective separation of the variables which are longitudinal and transverse with respect to the channel axis. The index of the transverse-quantization mode, n, is an adiabatic invariant, and the corresponding energy $\widetilde{E}_n(x)$ depends on the coordinate (x) along the channel axis. It appears as an effective potential in a one-dimensional Schrödinger equation describing the longitudinal motion of an electron. Those modes for which the maximum value of

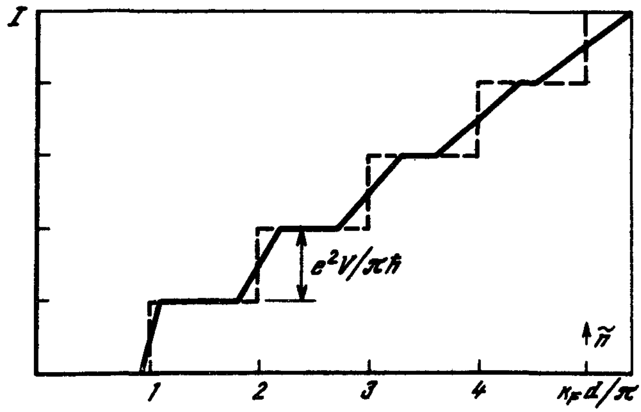

FIG. 1. Solid line—Current through the constriction versus the diameter at a finite extraction voltage V; dashed line—the case $V=0$.

the potential $\widetilde{E}_n(0)$ is below the Fermi energy contribute to the conductance. The extraction voltage applied to the contact, V, determines the band of energies eV near the Fermi level in which the electrons contribute to the current.

The nonlinearity in a ballistic conductance is usually determined by the small parameter $eV/\widetilde{E}_F$. A specific feature of a situation in which the value of d can be controlled is that as some channel n comes into play, the band of energies $E_F - \widetilde{E}_n(0)$ of the "current" states may be comparable to eV, even under the condition $eV/E_F \ll 1$. This circumstance is responsible for the pronounced nonlinearity near the constriction, which separates neighboring plateaus on the plot of $G(d)$.

In the present letter we show that at a finite voltage V the sharp steps (sharp in the limit $V \to 0$) on the current plot $I(d)$ become broader. The broadening increases linearly with the step index n, and at $n > 2E_F/eV$ the plateaus disappear from the plot of $I(d)$. On the plot of the differential conductance $G = \partial I/\partial V$ versus d, the integer

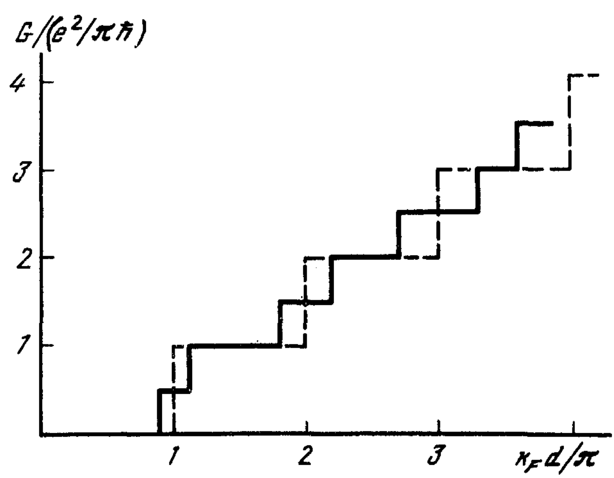

FIG. 2. The additional steps in the plot of $G(d)$ [see (4)].

plateaus are accompanied by some other plateaus, which lie between the integer plateaus (Figs. 1 and 2). The width of these additional plateaus increases with increasing eV and with increasing index n. For all of the additional plateaus (except for the first few) the values of $(\pi\hbar/e^2)G$ are approximately half-integers.

In the limit $V\to 0$, a given mode comes into play in the conductance as the quantity $E_F - \widetilde{E}_n(0)$ varies over the energy interval $\Delta_n = n\hbar^2/m(2Rd^3)^{1/2}$, where R is the radius of curvature of the constriction.[3] Let us find the component of the current from one mode for an arbitrary relation between eV and Δ_n. Assuming $eV/E_F \ll 1$ and $T \ll \Delta_n$, eV, and using the known formula[4] for the coefficient for transmission through a parabolic barrier, we find

$$\delta I_n = \frac{e}{\pi\hbar}\Delta_n \ln\left\{\frac{1+\exp\left(\dfrac{E_F-\widetilde{E}_n(0)+\dfrac{eV}{2}}{\Delta_n}\right)}{1+\exp\left(\dfrac{E_F-\widetilde{E}_n(0)-\dfrac{eV}{2}}{\Delta_n}\right)}\right\}. \qquad (1)$$

It can be seen from (1) that under the condition $eV \gg \Delta_n$ a given mode comes into play in the interval $|E_F - \widetilde{E}_n(0)| < eV/2$, and over essentially this entire interval the quantity δI_n is proportional to $E_F + eV/2 - \widetilde{E}_n(0)$. The current through the constriction is determined by the sum of partial currents (1) of the individual modes, and at $eV \gg \Delta_n$ we have

$$I(z) = \frac{e^2V}{\pi\hbar}\frac{(n_1+n_2)}{2} + \frac{eE_F}{\pi\hbar}(n_2-n_1)\left\{1-\frac{2(n_1^2+n_2^2+n_1n_2)+3(n_1+n_2)+1}{6z^2}\right\} - \frac{e^2\varphi_0}{\pi\hbar}(n_2-n_1),$$

$$n_1(z) = \left[z\left(1-\frac{eV+2e\varphi_0}{2E_F}\right)^{1/2}\right], \qquad n_2(z) = \left[z\left(1+\frac{eV-2e\varphi_0}{2E_F}\right)^{1/2}\right]. \qquad (2)$$

In place of the constriction diameter d here we have introduced the dimensionless variable $z = k_F d/\pi$. The square brackets in (2) mean the greatest integer. The potential $\varphi_0 = (\widetilde{E}_n(0) - E_n(0))/e$ determines the difference between the energy $\widetilde{E}_n(0)$ and the value[3] $E_n(0) = \hbar^2\pi^2 n^2/2md^2$, which arises because of the finite voltage V applied to the channel.

From (2) we easily see tht the nth plateau on the plot of $I(z)$ corresponds to an interval of z values for which we have $n_1 = n_2 = n$. The region of the nth step [which is the transition from the $(n-1)$th to the nth plateau] corresponds to those values of z for which we have $n_2 = n_1 + 1 = n$. The width of a step, δz_n, increases with its index: $\delta z_n = (eV/2E_F)n$. The middle of the nth step coincides with an integer value $z = n$ within a small quantity[2)] $\sim eV/E_F$. The picture drawn here holds for $z < \tilde{n} = [2E_F/$

$eV]$, (Fig. 1). As z increases, the $I(z)$ dependence approaches a linear dependence

$$I(z) = \frac{e^2 V}{\pi\hbar}(z - \frac{1}{2}) . \tag{3}$$

There are no plateaus at $z > \tilde{n}$, and the deviations from (3) do not exceed $\sim (eV/E_F)(e^2V/\pi\hbar)$ in magnitude. These deviations are described by a broken line with a typical change $\delta z \sim 1$. Note the term $-1/2$ in (3): It corresponds to the first quantum correction to the classical Sharvin formula.

The discussion above shows that not all of the states in the band of energies eV contribute equally to the current at values of z which are approximately integers. This circumstance is manifested in the unusual z dependence of the difference conductance, $G(z) = \partial I(z)/\partial V$. In differentiating (2) with respect to the voltage, we should allow for the implicit V dependence of φ_0 and d. The V dependence of φ_0 is determined by the particular distribution of the applied voltage along the channel. The $d(V)$ dependence stems from the electrostatic method by which the constriction is formed. It is caused by the repulsion of electrons out of the region under the gate. For definiteness, we assume that the electrostatic potential on the gate is zero. Correspondingly, the potentials on the left and right banks of the constriction are $V_G - V/2$ and $V_G + V/2$. We will show below that the implicity V dependence of φ_0 and d can be ignored in the region of greatest interest, $1 \ll z \lesssim n$, and the additional steps in the conductance (Fig. 2) correspond to half-integer values of the quantum:

$$G(z) = \frac{e^2}{\pi\hbar}(n - \frac{1}{2}), \qquad |z - n| \lesssim n\frac{eV}{4E_F} . \tag{4}$$

At small indices n, there are again some additional plateaus, but the corresponding values of G may be different from half-integers.

In analyzing the question of the potential distribution along the channel, we should first note that in a two-dimensional electron gas the electric field of the charge is screened over distances on the order of the first Bohr radius a_B (Ref. 5). As in the theory of classical point contacts,[6] we can thus make use of the condition of electrical neutrality. The deviation of φ_0 from zero in this case stems from the difference between the probabilities (W_- and W_+) that electrons will arrive from the left and right banks at the point of greatest narrowing. It is easy to see that we have $\varphi_0 \sim V(W_+ - W_-)/(W_+ + W_-)$. In the region $z \lesssim \tilde{n}$ we have a ratio $(W_+ - W_-)/(W_+ W_-) \sim 1/z$, since for a semiclassical barrier these probabilities differ for only a single mode (that which has come into play). Near the nth step we thus have $\varphi_0/V \sim 1/n$, and the contribution to G from the differentiation of φ_0 is small.

We would like to call attention to the effect of V on the value of d. We note at the outset that for a constriction of symmetric geometry we would have $\partial d/\partial V \propto eV/E_F$, and the corresponding contribution to G could be ignored. For an arbitrary geometry of the constriction we would have $\partial d/\partial V \lesssim \partial d/\partial V_G$. Determining the V_G dependence of the position of the boundaries of the constriction requires solving a nonlinear two-dimensional screening problem. We will be content with simply a parametric estimate of the value of $\partial d/\partial V_G$, assuming that the width of the lithographic gap in the gate[1,2] is equal to D. For the discussion below, the ratio a_B/D is an important small parameter. This parameter makes it possible to use a procedure of successive approximations to solve a self-consistent equation[5] in the two-dimensional case and to derive the estimate $\varphi^{(1)} \sim a_B V_G/(D-d)$ for the variations of the electric potential in the plane. A boundary of the two-dimensional gas evidently corresponds to the value $e\varphi^{(1)} \doteq E_F$. We thus find

$$D - d \sim \frac{eV_G}{E_F} a_B . \qquad (5)$$

From (2) and (5) we can find an estimate of the contribution of the dependence $d(V)$ to the value of the conductance. In units of the quantum, this contribution does not exceed $a_B k_F/n$. For the heterostructures used in Refs. 1 and 2 we would have $a_B k_F \sim 1$, and at large values of n the estimated contribution would be insignificant. Consequently, relation (4) holds for most of the additional plateaus.

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[1)]The diameter is controlled by the gate voltage V_G.
[2)]This assertion is based on the inequality $\varphi_0 < V/n$, which we will derive below.

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