Coherent amplification of an ultrashort pulse in a three-level medium without a population inversion

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It is predicted that an ultrashort pulse in a three-level medium with Λ configuration will be amplified in the absence of a population inversion and coherent optical pumping. This effect occurs as a result of population capture in the two lower levels due to the excitation of the low-frequency coherence.

1. Let us consider the coherent propagation of an ultrashort pulse in a three-level medium of Λ configuration (Fig. 1), when two ground-state sublevels are optically coupled with the upper level. We assume that the spectral width of the homogeneous optical transitions is greater than the spectral width of the pulse, which in turn overlaps the splitting frequency: $T_2^{-1} \gg \tau_p^{-1} \gg \omega_{21}$. Under these conditions the ultrashort pulse interacts resonantly simultaneously with the two optical transitions. This situation differs fundamentally from the familiar case of the propagation of a simulton in a three-level medium,[1] which occurs under directly opposite conditions: $T_2^{-1} \ll \tau_p^{-1} \ll \omega_{21}$ and which is described by the equations

$$
\begin{aligned}
\partial u/\partial \zeta &= -2\eta n - (1+\eta^2)u\,,\\
\partial n/\partial \zeta &= -3[(1+\eta^2)n - (1-\eta^2)\tilde{n} + 2\eta u]\,,\\
\partial \tilde{n}/\partial \zeta &= (1-\eta^2)n - (1+\eta^2)\tilde{n}\,,\\
\partial I/\partial z + c^{-1}\partial I/\partial t &= -\sigma N I[(1+\eta^2)n - (1-\eta^2)\tilde{n} + 2\eta u]\,.
\end{aligned} \tag{1}
$$

These equations are derived from the equation for the density matrix ρ in the approximation of the adiabatic summation of the polarization in the optical transitions beyond

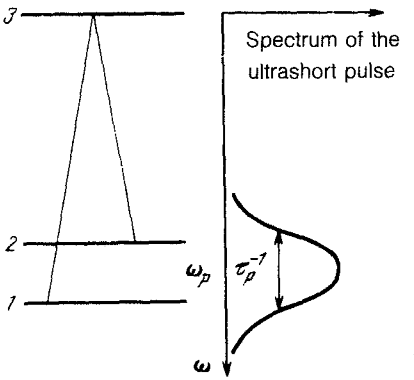

FIG. 1. Schematic diagram of the resonant interaction of a three-level medium of Λ configuration with an ultrashort pulse which overlaps the splitting frequency ω_{21} in the spectrum.

the field E for an ultrashort pulse of length τ_p, which is much shorter than the population relaxation times.[2] These equations describe the transmission of the radiation intensity $I = c|E|^2/8\pi\hbar\omega_p$ through a three-level medium, with allowance for the low-frequency coherent effects. Here N is the concentration of atoms, $\eta = \mu_{32}/\mu_{31}$ is the ratio of the dipole moments, $\zeta = \sigma\int_{-\infty}^{t} I(t')dt'/2$, ω_p is the carrier frequency of the pulse, $\sigma = 4\pi\mu_{31}^2 T_2\omega_p/c\hbar$, $n = (\rho_{22} + \rho_{11} - 2\rho_{33})/2$, $\tilde{n} = (\rho_{22} - \rho_{11})/2$, $u = \mathrm{Re}\rho_{21}$, and ρ_{21} is the low-frequency coherence.

2. The general solution of the first three equations in (1), which describes the conversion of the parameters of the medium into the field of an ultrashort pulse, is

$$\begin{pmatrix} u \\ n/\sqrt{3} \\ \tilde{n} \end{pmatrix} \equiv \mathbf{x} = \sum_{\alpha=1}^{3} C_\alpha \mathbf{x}^{(\alpha)} e^{-\lambda_\alpha \zeta}\ ; \quad C_\alpha = \sum_{j=1}^{3} x_{0j} x_j^{(\alpha)}, \quad \mathbf{x}_0 \equiv \begin{pmatrix} u_0 \\ n_0\sqrt{3} \\ \tilde{n}_0 \end{pmatrix}. \tag{2}$$

Here $\mathbf{x}_0$ is the vector of the state of the medium before the arrival of an ultrashort pulse, $\lambda_1 = 0$, $\lambda_2 = 4(1+\eta^2)$, $\lambda_3 = 1+\eta^2$,

$$\mathbf{x}^{(1)} = \frac{\sqrt{3}}{2}\begin{pmatrix} -2\eta/(1+\eta^2) \\ 1/\sqrt{3} \\ (1-\eta^2)/(1+\eta^2) \end{pmatrix}, \quad \mathbf{x}^{(2)} = \begin{pmatrix} -\eta/(1+\eta^2) \\ -\sqrt{3}/2 \\ (1-\eta^2)/2(1+\eta^2) \end{pmatrix},$$

$$\mathbf{x}^{(3)} = \begin{pmatrix} (1+\eta^2)/(1+\eta^2) \\ 0 \\ 2\eta/(1+\eta^2) \end{pmatrix}; \quad \mathbf{x}^{(\alpha)}\mathbf{x}^{(\beta)} = \delta_{\alpha\beta}. \tag{3}$$

Substitution of solution (3) in the equation for the intensity (1) gives

$$\frac{\partial I}{\partial z} + c^{-1}\frac{\partial I}{\partial t} = \frac{1}{2}\sigma N I \lambda_2 C_2 e^{-\lambda_2\zeta},$$

$$\lambda_2 C_2 = -4\eta u_0 - 2(1+\eta^2)n_0 + 2(1-\eta^2)\tilde{n}_0. \tag{4}$$

In the special case in which one of the optical transitions is forbidden, i.e., $\eta = 0$ or $\eta = \infty$, Eq. (4) is the same as the equation for the transport of radiation intensity in a two-level, inertia-free medium in a balanced approximation.[3,4] The solution of this equation describes the extensively studied processes of amplification and absorption of ultrashort pulses in the inverted and noninverted two-level media, respectively. Equation (4) generally has a similar solution:

$$I(t,z) = \frac{I_0(t - z/c)}{1 - \{1 - \exp(-\sigma N\lambda_2 \int_0^z C_2 dz/2)\}\exp[-\lambda_2\zeta_0(t - z/c)]},$$

$$\zeta_0(t) = \frac{\sigma}{2}\int_{-\infty}^{t} I(t')dt', \tag{5}$$

where $I_0(t)$ is the intensity of the ultrashort pulse at the entrance to the three-level medium ($z=0$).

3. In a three-level medium the amplification factor is determined, according to (4), by C_2 and depends not only on the difference in the populations in the optical transitions but also on the low-frequency coherence. Because of this situation, an ultrashort pulse can be amplified, despite the absence of inversion and coherent optical pumping in each one of the optical transitions. Amplification in a two-level medium is, as we know, impossible under these conditions.[4,5] An inversion-free amplification occurs over a broad range of parameters of the medium:

$$C_2 > 0, \quad 0 < \rho_{33}^{(0)} \leqslant \rho_{11}^{(0)}\rho_{22}^{(0)}\ ; \quad \mathrm{Sp}(\rho^{(0)})^2 \equiv \sum_{i=1}^{3}\rho_{ii}^{(0)2} + 2|\rho_{21}^{(0)}|^2 \leqslant 1. \tag{6}$$

This region is illustrated in Fig. 2 for the same dipole moments in each optical transition when $\mathrm{Im}\rho_{21}^{(0)}=0$ and $\tilde{n}_0=0$. The maximum amplification in this region is attained at $\rho_{33}^{(0)}=1/3$ and $u_0=-1/\sqrt{3}$. It is only $\sqrt{3}$ times lower than that in a completely inverted medium.

It should be emphasized, on the other hand, that even a slight filling of the upper level ($\rho_{33}^{(0)} \ll 1/3$) as a result of maximum excitation of the low-frequency coherence $-u_0 \approx \sqrt{\rho_{11}^{(0)}\rho_{22}^{(0)}}$, produces, according to (4), the same amplification as the inverted region with an inversion of $\rho_{33}^{(0)}$ (Fig. 3).

Physically, inversion-free amplification is associated with the existence in the three-level medium of a coherent state of the lower sublevels $C_1\mathbf{x}^{(1)}$. Being in this state, the atoms do not interact with the field because of the interference of the 1–3 and 2–3

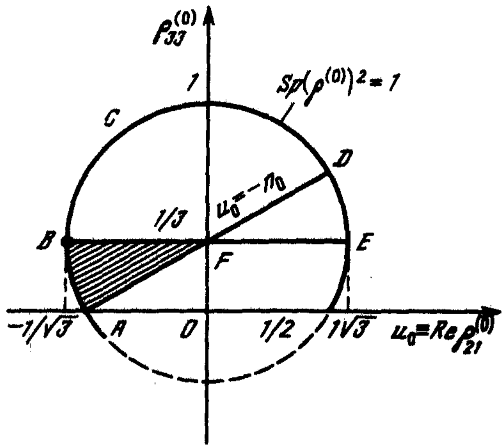

FIG. 2. The plane of the initial states of the medium in the case $\rho_{11}^{(0)}=\rho_{22}^{(0)}$, $\mathrm{Im}\rho_{21}^{(0)}=0$. The letters denote the region of inversion-free amplification (ABF), the region of amplification in the medium with an inversion (BCDF), and the region of absorption in the medium with an inversion (DEF).

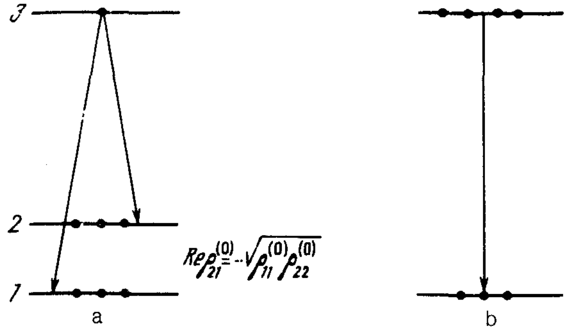

FIG. 3. Inversion-free amplification factor in a three-level medium (a) is the same as that in the inverted two-level medium (b).

transitions.[2,6,7] Excitation of low-energy coherence ($u_0 \neq 0$) at the time of the arrival of the pulse partially transfers the atoms to the indicated state of coherent population capture, thereby ostensibly taking out of commission a fraction of the atoms which are situated in the lower sublevels but which are incapable of absorbing the field. As a result, the ultrashort pulse empties the upper level: $\rho_{33} - \rho_{33}^{(0)} = -C_2[1 - \exp(-\lambda_2 \zeta)]$ even when its population is lower than the populations of the lower levels. The energy drawn from the medium is converted to pulse energy: $W_p(z) = W_p(0) + \hbar\omega_p N \int_0^z (\rho_{33} - \rho_{33}^{(0)}) dz$.

A similar inversion-free amplification is possible in several other cases. In particular, it occurs every time the population of the upper level in the corresponding coherent-capture state is, as a result of low-frequency coherence excitation, lower than that in the initial state of the medium. Radiation removes this population "excess," striving to transform the medium to a state of coherent population capture at the lower sublevels. For this reason, it intensifies even in an uninverted medium. In Particular, it can be shown that an inversion-free amplification is possible even under conditions (typical in the case of resonance stimulated Raman scattering) of propagation in a three-level medium of two quasimonochromatic radiation components with the carrier frequencies ω_{31} and ω_{32}.

4. Excitation of coherence ρ_{21} can be realized by means of a resonant microwave field, for example, in the form of a $\pi/2$ pulse, which transforms the medium to a state with equal populations of levels 1 and 2 and with a maximum value of Im ρ_{21}. Since the amplification of the ultrashort pulse is proportional to Re $\rho_{21}^{(0)}$, the optical pulse lags behind the microwave pulse by one-fourth or three-fourth of a period $2\pi/\omega_{21}$ (depending on the sign of the difference in the populations in the 1–2 transition before the arrival of the microwave pulse). According to the law of conservation of the Bloch vector, the excitation of the coherence increases as this difference is increased. To increase the coherence, it would be desirable to deliberately empty one of the sublevels, for example, by a selective use of resonance radiation with respect to frequency or polarization or by cooling the medium.

A partial filling of the upper level, which is necessary for amplification, can be achieved by incoherent optical pumping or by other conventional methods. In the infrared and submillimeter ranges an inversion-free amplification can be achieved through a thermal equilibrium filling of the upper level.

An inversion-free amplification can be seen experimentally, for example, in sodium vapor or in ruby. The predicted effect can be used effectively to produce amplification or excitation in those transitions where it is difficult to achieve population inversion.

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