Nonlinear pairing of light and dark optical solitons

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Nonlinear pairing of light and dark optical solitons is shown to be possible as a result of phase cross-modulation of the waves in a nonlinear dispersive medium. On this basis fundamentally new methods can be developed for a nonlinear stabilization of high-power wave packets in the region of positive and negative dispersion of the group velocities of the interacting waves.

The idea that bound states can be produced by particle pairs—pairing—is one of the fundamental ideas of modern physics. The extension of this idea to nonlinear optics leads, as we will show in this letter, to the theoretical prediction that new "quasiparticles" are produced as a result of nonlinear pairing of optical solitons of various wavelengths ("colors"). The pairing of light and dark optical solitons which we are considering here raises a fundamentally new possibility of nonlinear stabilization of intense supershort light pulses and production of optical solitons in the region of positive and negative dispersion of group velocities as a result of phase cross-modulation of the interacting waves.[1,2]

The nonresonant self-effect of N waves in a nonlinear cubic dispersive medium is described by a system of Schrödinger equations for the complex amplitudes of interacting waves:

$$2ik_m\left(\frac{\partial E_m}{\partial z}+\frac{1}{v_m}\frac{\partial E_m}{\partial t}\right)=-k_m\frac{\partial^2 k_m}{\partial\omega_m^2}\frac{\partial^2 E_m}{\partial t^2}+\frac{k_m n_2|E_m|^2E_m}{n_0}$$

$$+\frac{2k_m n_2}{n_0}\sum_{n=1,\,n\neq m}^{N}|E_n|^2E_m. \qquad (1)$$

This system, in which standard notation is used, describes the following physical processes: the competition between dispersion effects and nonlinear effects—the self-effect of waves and the "reactive" interaction (without energy transfer) of waves due to the phase cross-modulation. The phase cross-modulation causes the self-effect of the wave packets to change qualitatively; in particular, it causes the formation of bound states of optical solitons with different wavelengths—soliton pairing. The principal physical mechanisms for nonlinear soliton pairing can be illustrated, on the basis of model (1), by using the example of the interaction of two waves:

$$i\partial\Psi_1/\partial z=1/2\partial^2\Psi_1/\partial\tau^2+R_{11}|\Psi_1|^2\Psi_1+R_{12}|\Psi_2|^2\Psi_1, \qquad (2)$$

$$i(\partial\Psi_2/\partial z+\nu\partial\Psi_2/\partial\tau)=1/2\partial^2\Psi_2/\partial\tau^2+R_{12}|\Psi_1|^2\Psi_2+R_{22}|\Psi_2|^2\Psi_2. \qquad (3)$$

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Equations (2) and (3) are written in the associated coordinate system in standard dimensionless variables: $\Psi_i = E_i/E_{oi}$; $\tau = (t - z/v)/\tau_{0i}$; $z = z/z_d$, where z_d is the length of the dispersive spreading of pulses, and R_{ij} are the nonlinearity parameters. In the absence of a phase cross-modulation ($R_{ij} = 0$,) Eqs. (2) and (3) degenerate to a $i \neq j$ nonlinear Schrödinger equation, whose partial solutions are the light solitons—particles ($R_{ij} > 0$) and dark solitons—holes ($R_{ij} < 0$): $\Psi_1^+(z,\tau) = \kappa/\sqrt{R_{11}}$ sech$(\kappa\tau)$ $\exp(-i\kappa^2 z/2)$; $\Psi_2^-(z,\tau) = \kappa/\sqrt{|R_{22}|}$, tanh $(\kappa\tau)\exp(i\kappa^2 z)$. Equations (2) and (3) contain the first integral which expresses the law of conservation of the total number of soliton-particles and soliton-holes $\int|\Psi_1^+|^2 d\tau + \int(|\Psi_2^-|^2 - |\Psi_2^-(z = \pm\infty)|^2)d\tau$ $= N_1^+ + N_2^-$. The Hamiltonian of the system of interacting solitons can, on the basis of model (2), (3) for $R_{12} = R_{21}$, be written as follows: $H = H_{01} + H_{02} + H_{\rm int}$, where $H_{oi} = \int(|\Psi_{i\tau}|^2 - R_{ii}|\Psi_i|^4)d\tau$ is the Hamiltonian of "free" solitons in the absence of interaction, and the Hamiltonian of the pairwise interaction of solitons, $H_{\rm int}$, can be written in the form: $H_{\rm int} = \int\Psi_1^*\Psi_2^* R_{12}\Psi_1\Psi_2$, where R_{12} is useful to treat as the interaction energy of a soliton pair. The condition $R_{12} = R_{21}$ means that the bound states of different colors in a system of interacting waves, (1)–(3), appear and disappear in pairs.

The system of equations (2) and (3) has the following soliton solutions: 1) paired light solitons (all $R_{ij} > 0$) and paired dark solitons (all $R_{ij} < 0$) situated in the parametric region $R_{12} = 1 - R_{11}$ and $R_{21} = 1 - R_{22}$ for $0 < |R_{ii}| < 1$. Solutions such as "light + light" soliton were obtained for the first time by Manakov[3]; 2) paired light and dark solitons without an inversion of wavelengths (colors). In the absence of interaction ($R_{12} = R_{21} = 0$) the light solitons ($R_{11} > 0$) and dark solitons ($R_{22} < 0$) are the true solitons of the nonlinear-Schrödinger-equation model. An exact solution of (2) and (3) for $R_{ij} \neq 0$, $R_{1j} > 0$, and $R_{2j} < 0$ is a quasiparticle comprised of paired $i \neq j$ light and dark solitons:

$$\Psi_1^+(z, \tau) = \text{sech}\, \tau e^{-i(1/2 + |R_{12}|)z}; \quad \Psi_2^-(z, \tau) = \tanh\, \tau e^{i|R_{22}|z}. \tag{4}$$

This solution applies to the region of the parameters $R_{12} = R_{11} - 1$; $|R_{21}| = |R_{22}| - 1$. Since $R_{1j} > 0$, we see that these conditions determine only the relationship between the parameters R_{ij} but do not impose any constraints on the limits of their variation. Since the parameters $R = N^2$ are related to the number of solitons of the same color in each pulse, the dark multisoliton pulse in this system stabilizes nonlinearly the light multisoliton pulse (Fig. 1). The physical explanation of the wave packet stabilization consists in transposing the frequency modulation of one wave to another, so that the positive frequency chirp in the light pulse is exactly cancelled by the negative frequency chirp in the dark pulse. Using the space-time analogy, this effect can be viewed as a mutual self-focusing of beams at different wavelengths. A beam with a lower intensity at the axis is a nonlinear lens extending across the medium, in which a waveguide propagation of the second beam occurs at a power level many times greater than the critical power level of the self-focusing. This situation closely resembles Askar'yan's[4] "banana self-focusing"; 3) light and dark paired solitons with a color inversion. Let us invert the wavelengths of the interacting solitons; i.e., let us determine whether light

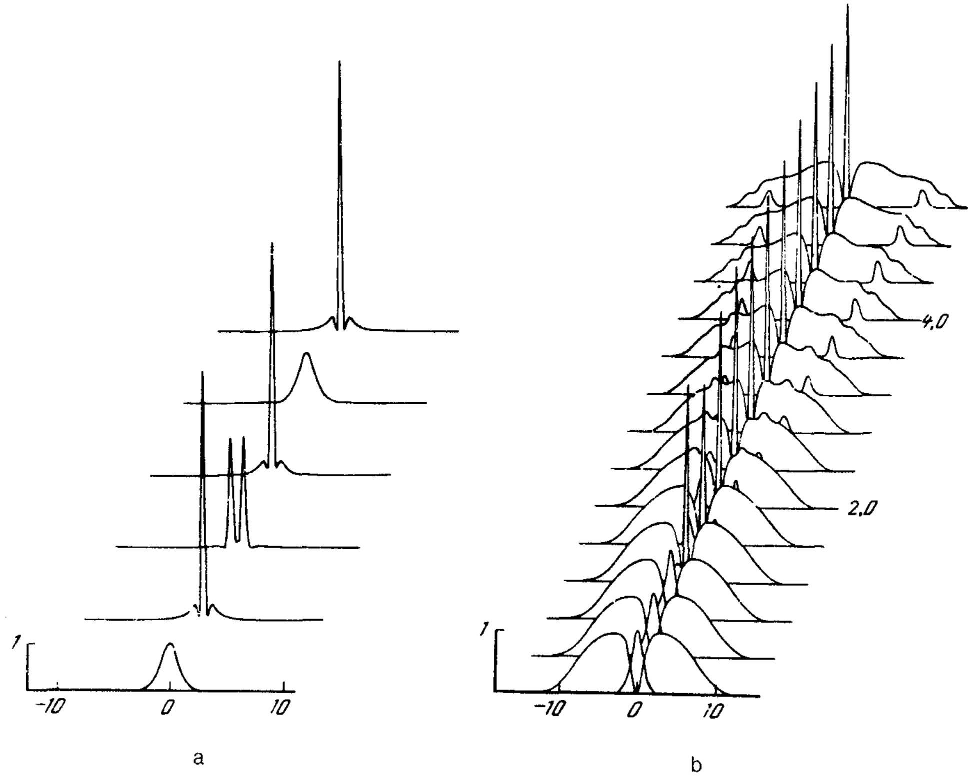

FIG. 1. (a) Dynamics of the bound state $N^+ = 3$ $(R_{11} = 9)$ of light Schrödinger solitons in the absence of interaction with the dark solitons. (b) Nonlinear stabilization of a multisoliton pulse in the bound state with a dark soliton (noninverted case 2) $R_{11} = (N_1^+)^2 = 9$; $R_{12} = 8$; $R_{22} = -9$; $R_{21} = -8$. $\Psi_1^+(z=0,\tau) = \operatorname{sech}\tau$.

solitons with $R_{ij} < 0$ can exist in the bound state with a dark soliton with $R_{ij} > 0$. In the absence of interaction such states theoretically cannot occur. Soliton pairing in the phase cross-modulation of the waves makes it possible to obtain such a non-trivial solution of (2) and (3)—optical "superfluidity" effect:

$$\Psi_1^-(z,\tau) = \operatorname{sech}\tau e^{-(1/2-|R_{21}|)z};\quad \Psi_2^+(z,\tau) = \tanh\tau e^{-i|R_{22}|z}. \tag{5}$$

The conditions under which solutions (5) hold are: $|R_{12}| = |R_{11}| + 1$ and $|R_{21}| = |R_{22}| + 1$. As in case 2), these conditions do not impose any constraints on the limits of variation of the parameters R_{11} and R_{22}, which are related to the number of solitons in the pulses $R_{ii} = N_i^2$. The fact that a soliton-like solution is obtained in the region of positive dispersion $(R_{11}, R_{12} < 0)$ is illustrated in Fig. 2 by the results of a numerical solution of Eqs. (2) and (3). Since the parameter $R_{ii} = 1$ corresponds to a one-soliton solution of the nonlinear Schrödinger equation, a transition from the non-inverted case 2) to the inverted case 3) occurs each time a quantity $\Delta R = 2$, which is

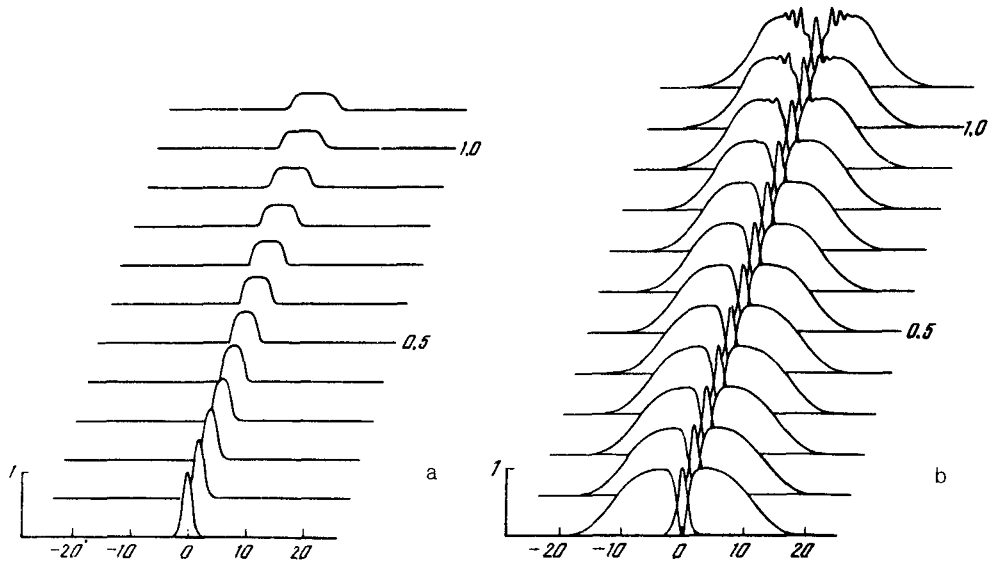

FIG. 2. (a) Dynamics of the self-effect of the temporal envelope of the wave packet in the region of positive dispersion of the group velocities ($R_{ij} < 0$) in the absence of interaction with the dark solitons, calculated on the basis of the nonlinear-Schrödinger-equation model for $R_{22} = -9$; (b) formation of the bound state of a light soliton in the region of negative dispersion of the group velocity (inverted case 3) $R_{11} = -9$; $R_{12} = -10$; $R_{21} = 10$; $R_{22} = 9$. $\Psi_1^-(z = 0,\tau) = \operatorname{sech}\tau$; $\Psi_2^+(z = 0,\tau) = \tanh\tau\exp(-\tau^4/10)$; $R = N^2$, where N is the number of solitons per pulse.

equal to the energy of two solitons, is added to the "binding energy" R_{12}. The fact that light solitons can exist in the region $\partial^2 k/\partial\omega^2 > 0$, in the case of phase cross-modulation of waves and tanh modulation of the fundamental wave was pointed out in Refs. 5 and 6. In the numerical experiments we studied the effect on the bound state of the solitons of the dispersive spreading of the carrier pulse with $R_{2j} < 0$ (see Refs. 7 and 8), of the development of a modulational instability in it at $R_{2j} > 0$, of the role of a temporal mismatch of the light and dark solitons, and of the mismatching of the group velocities. The stability of the solutions obtained by us to the temporal mismatching Δ is determined by the nature of the soliton interaction $H_{\rm int} = \int|\Psi_1|^2 R_{12}|\Psi_2|^2 d\tau = A_0 + A_1\Delta + A_2\Delta^2$ at low values of Δ. It can easily be shown that $A_1 = 0$ and that the nature of the interaction (attraction of repulsion) is governed by A_2. A calculation shows that solutions 1) and 3) coprrespond to attraction and solution 2) corresponds to repulsion.

The inverted case 3) is especially attractive when a nonlinear pairing of optical solitons is achieved experimentally, since in this case both the relation $R_{123} = 2$ with $R_{11} = 1$, which is characteristic for single-mode optical waveguides, and the stability against transient pulse mismatching can be achieved (Fig. 3). At $\lambda_0(\partial^2 k/\partial\omega^2 = 0) = 1.31\ \mu$m a soliton propagation regime of a pulse at the wavelength of 1.06 μm of the same length and intensity can be achieved by changing the conditions of the pioneering experiment[9] on the observation of light solitons in lightguides, i.e., by form-

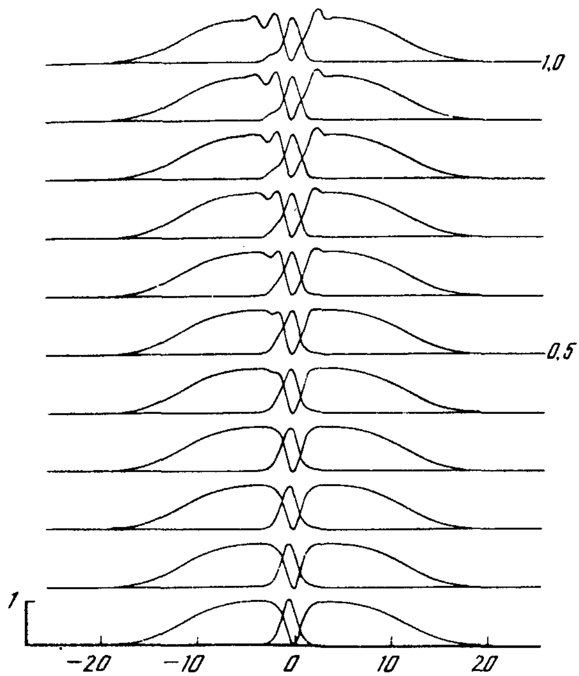

FIG. 3. Stability of the bound state of the light soliton and the dark soliton with a color inversion (case 3) against a temporal mismatch of the envelopes. $R_{11} = -1$; $R_{12} = -2$; $R_{22} = 1$; $R_{21} = 2$, $\Psi_1^-(z=0,\tau) = \text{sech}(\tau+\Delta)$; $\Delta = 0.5$; $\Psi_2^+(z=0,\tau) = \tanh\tau\exp(-\tau^4/10)$.

ing a dark soliton at the wavelength $\lambda = 1.56$ μm, whose duration would be $\tau = 7$ ps and whose intensity of the carrier pulse would be $I = 1.2$ W. In multimode lightguides soliton pairing is governed by conditions (4) and (5).

In summary, nonlinear optical soliton pairing in the phase cross-modulation of waves raises the possibility of developing fundamentally new methods of nonlinear stabilization of high-power wave packets.

After this paper had been prepared for publication a paper by S. Trillo, S. Wabnitz, E. M. Wright, and G. I. Stegeman, in which they obtained solution (5), was published in the October 1988 issue of Optics Letters.

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