Intraband tunneling flip of quasimomentum in connection with the dynamics of Josephson junctions

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(Submitted 10 November 1988)
Pis'ma Zh. Eksp. Teor. Fiz. **48**, No. 11, 630–632 (10 December 1988)

Intraband tunneling flip of quasimomentum, $p \to -p$, is analyzed. Such processes are important for the dynamics of Josephson junctions. They contribute substantially to relaxation processes.

The dynamics of a Josephson junction is recognized as being similar to the dynamics of a quantum-mechanical particle which is interacting with a medium in a sloping periodic potential $V = -E_J \cos 2\varphi - I\varphi/e$ (Refs. 1–3). If the energy of the Josephson junction, $E_J = I_c/2e$, is large in comparison with the frequency of plasma oscillations, $\Omega_p = 2e(E_J/C)^{1/2}$, where C is the capacitance of the junction, the particle moves in a narrow energy band, and the single-band approximation is sufficient.

In addition to the ordinary scattering, the interaction with the medium causes a tunneling flip of the quasimomentum, $p \to -p$, within a single band.

A process of this sort is analogous to the tunneling of a particle through a potential barrier; the quasimomentum is playing the role of the coordinate. As we will see below, a tunneling flip of quasimomentum has an important effect on the dynamics of Josephson junctions, contributing to the relaxation mechanism.

To calculate the probability for a tunneling flip of quasimomentum, we examine the partition function of the system consisting of a particle which is interacting with a medium:

$$Z=\int DpD\varphi \exp\left\{\int_0^{1/T} d\tau\left[-\epsilon(p)+ip\frac{\partial\varphi}{\partial\tau}-\frac{\eta}{4\pi}\int_{-\infty}^{\infty} d\tau'\left(\frac{\varphi(\tau)-\varphi(\tau')}{\tau-\tau'}\right)^2\right]\right\}. \tag{1}$$

Here p is the quasimomentum of the particle, and $\epsilon(p)$ is the energy spectrum in the single-band approximation, given by

$$\epsilon(p)=-\frac{\delta}{2}\cos\pi p \quad \delta=16\left(\frac{E_J\Omega_p}{\pi}\right)^{1/2}\exp\left(-\frac{8E_J}{\Omega_p}\right). \tag{2}$$

The viscosity coefficient is $\eta=1/Re^2$.

The integration over one of the p's in path integral (1) must be carried out within the Brillouin zone, $|p|<1$.

Integrating over $\varphi(\tau)$ in (1), we find an expression for the partition function which depends on the variable $p(\tau)$ alone. Within a normalization we have

$$Z=\int Dp\exp\left\{-\int_0^{1/T} d\tau\left[\epsilon(p)+\frac{1}{4\pi\eta}\int_{-\infty}^{\infty} d\tau'\left(\frac{p(\tau)-p(\tau')}{\tau-\tau'}\right)^2\right]\right\}. \tag{3}$$

Expression (3) is exactly the same as the expression for the partition function of a massless particle which is moving in a potential $\epsilon(p)$ and which is interacting with a heat reservoir.[1-3]

We know that in this case there can be a tunneling of the particle through the potential barrier $\epsilon(p)$ and that the lifetime of the particle in one of the potential wells of $\epsilon(p)$ becomes finite.

In contrast with the standard formulation of the problem of calculating the total lifetime of the particle in the potential well, in the case at hand we need to find the tunneling probability for a given initial value of the quasimomentum p. The method of instantons cannot be used to solve this problem.

To calculate the probability for a tunneling breakdown for a given initial value of the quasimomentum p, we make use of the analogy, which we mentioned above, between the system of interest here and a massless particle which is interacting with a medium. The role of the medium can be played by an infinite set of harmonic oscillators.[1] At values of the energy $\epsilon(p)$ which are close to the top of energy band (2), the multidimensional quantum-mechanical problem of calculating the probability for a tunneling from state p to $2-p$ can be solved exactly. The result is

$$\Gamma_S(p,T)=B\exp[-w(p,T)], \tag{4}$$

where

$$w=\left[1-\frac{2\epsilon(p)}{\delta}\right]\left(\frac{T}{\delta}+\frac{\pi\eta}{4}\right)^{-1},$$

$$B=\frac{2\omega_c^2}{\pi^3\delta\eta^{3/2}}\left[1-\frac{2\epsilon(p)}{\delta}\right]^{1/2}. \tag{5}$$

The cutoff frequency ω_c in (5) is equal in order of magnitude to the plasma-oscillation frequency Ω_p.

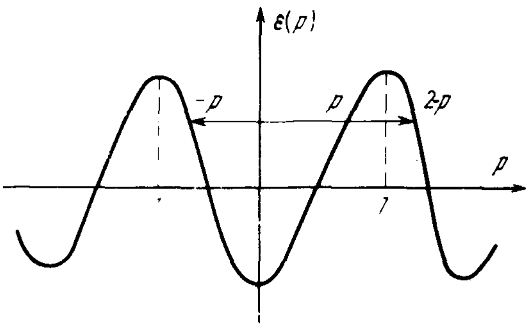

FIG. 1.

At a semiclassical accuracy level, the energy $\epsilon(p)$ is conserved during intraband tunneling, while the quasimomentum p in the band is replaced by $-p$ (Fig. 1). Although intraband tunneling processes are exponentially small, on the order of the parameter $1/\eta$, the coefficient of the exponential function in (4) is nevertheless large, and processes of this sort can compete with ordinary relaxation, whose probability is proportional to $\eta\delta$.

The mechanism for intraband tunneling discussed above takes on special importance for the dynamics of Josephson junctions, in which the dissipation stems from a tunneling of normal electrons. The equation for the distribution function in such junctions is

$$\left(\frac{\partial}{\partial t}+\frac{I}{e}\frac{\partial}{\partial p}\right)n(p)$$
$$=2\eta\epsilon(p)\left\{-n(p-\operatorname{sgn}p)-n(p)+[n(p-\operatorname{sgn}p)-n(p)]\coth\frac{\epsilon(p)}{T}\right\}-\Gamma(p)[n(p)-n(-p)]. \quad (6)$$

Aside from the last term, this equation is the same as that in Refs. 4–6. The last term on the right side of Eq. (6) stems from intraband tunneling and consists of two parts: $\Gamma=\Gamma_S+\Gamma_T$, where Γ_S is given by (4) and corresponds to the dissipation mechanism which stems from the normal resistance shunting the junction. The component which stems from the tunneling of normal electrons, Γ_T, requires a separate calculation. Oscillations in the voltage at frequencies which are multiples of $2\pi I/e$ are thus damped even if the shunting resistance is infinite.

We wish to thank A. I. Larkin for a useful discussion of these results.

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Translated by Dave Parsons