Dissipative amplification of flute vortices in a plasma

V. P. Pavlenko, V. I. Petviashvili, and V. B. Taranov
Institute of Nuclear Research, Academy of Sciences of the Ukrainian SSR

(Submitted 30 November 1988)
Pis'ma Zh. Eksp. Teor. Fiz. **49**, No. 1, 10–12 (10 January 1989)

Two-dimensional vortices which move at a velocity below the ion drift velocity can exist in the stability region of flute waves. Such vortices may be amplified by a dissipation due to ions. As a result, convection in the plasma will be intensified.

Flute waves in an ideal plasma may be either stable or unstable, depending on the sign of the curvature of the magnetic field lines.[1] They go into a very nonlinear regime even at a comparatively small amplitude. A self-localization of wave packets into solitary vortices, with a large number of trapped particles, occurs.[2] In an unstable ideal plasma these vortices move at a velocity higher than the ion drift velocity, while in a stable plasma they move more slowly. In the present letter we show that slow vortices may be amplified when there is a viscosity. It follows that, even in the stability region of an ideal plasma, flute waves can contribute substantially to an anomalous transport of heat and particles through a vortex convection.

A system of model equations for the relative perturbation of the density n and the electric potential ϕ was proposed in Ref. 2 for describing the evolution of flute waves in an ideal plasma. When viscosity is taken into account, that system of equations

 0021-3640/89/010009-04$01.00

becomes

$$\frac{\partial n}{\partial t} + \frac{\kappa c}{B_0}\frac{\partial \phi}{\partial y} = \frac{c}{B_0}J(n, \phi), \tag{1}$$

$$\frac{c}{B_0\Omega_i}\left\{\frac{\partial}{\partial t} + v^*\frac{\partial}{\partial y}\right\}\psi - v_0\frac{\partial n}{\partial y} - \mu\frac{c}{B_0\Omega_i}\Delta\Delta U = \frac{c^2}{B_0^2\Omega_i}\left\{J(\psi, \phi) - \frac{T_i}{e}\mathrm{div}J(n, \nabla\phi)\right\}, \tag{2}$$

where

$$\psi \equiv \Delta\phi;\ U \equiv \phi + \frac{T_i}{e}n;\ J(n, \phi) = \frac{\partial n}{\partial x}\frac{\partial \phi}{\partial y} - \frac{\partial n}{\partial y}\frac{\partial \phi}{\partial x};$$

$$\Omega_i = \frac{eB_0}{m_i c},\ \kappa = -\frac{d\ln n_0}{dx} > 0;\ v^* = -\frac{\kappa T_i}{m_i\Omega_i},\ \text{and}\ v_0 = \frac{g}{\Omega_i}$$

are the velocities of the diamagnetic and magnetic ion drift, respectively; and μ is the ion viscosity coefficient, which was derived in Ref. 3. The magnetic field B_0 is directed along the z axis; the plasma and this field are nonuniform along the x axis; and the derivatives along the magnetic field are assumed to be negligibe. This relatively simple form of the equations has been achieved thanks to the assumption that a self-localization of flute perturbations is possible. That assumption makes it possible to incorporate the spatial variations in the plasma and the magnetic field by means of the constants κ and g.

In the absence of a viscosity ($\mu = 0$), system (1), (2) conserves the energy integral

$$W = \int\left\{(\nabla\phi)^2 - \left(\frac{B_0}{\kappa c}\right)^2\kappa g n^2\right\}dxdy. \tag{3}$$

Furthermore, regardless of the viscosity, this system conserves the functional series of integrals

$$\int\{f(n-\kappa x) - f(-\kappa x)\}dxdy,$$

where f is an arbitrary function.

Steady-state solutions of Eqs. (1), (2) in the form of dipole vortices, traveling at a constant velocity and solitary dipole vortices, similar to the Rossby-wave vortices found in Ref. 4, were derived in Ref. 2 for the case without viscosity. At infinity these solutions decay as $\exp(-r/a)$, where $r^2 = x^2 + (y-ut)^2$, and

$$a^2 = \frac{\kappa g}{u(u-v^*)}. \tag{4}$$

We see that in a stable plasma ($\kappa g < 0$) the vortex localization condition ($a^2 > 0$) holds for vortices which are moving at a velocity lower than the ion drift velocity. In an unstable plasma ($\kappa g > 0$) this condition holds for vortices which are moving faster than the ion drift velocity and also for vortices which are moving in the direction opposite the ion drift.

In the present study we have used numerical methods to examine the stability problem and the effect of a viscosity on the solutions which have been found. We solved Eqs. (1), (2) by the Lax-Wendroff method (variable directions with periodic boundary conditions on a 64×64 mesh). To calculate the spatial derivatives, we used a fourth-order-accuracy scheme. We used fast Hartley transforms to invert the Laplacian. As an initial condition we adopted a profile approximating the exact steady-state solution in the form of a dipole vortex.[2,4]

The numerical calculation shows that in a structurally unstable plasma the vortices are unstable even in the absence of a viscosity, and they break up over a time on the order of the reciprocal of the growth rate of the linear flute instability. A calculation for a stable plasma showed that under the condition $\mu = 0$ solitary vortices of the type in Refs. 2 and 4 do not decay; in fact, they retain their shape. At $\mu > 0$, by virtue of the initial condition $n = (\kappa c/uB)\phi$, we have $U \sim [1 - (v^*/u)]\phi$, and the effective viscosity in (2) changes sign, since we have $v^*/u > 1$ in a stable plasma, according to (4). This effect is analogous to the change in the sign of Landau damping in the "universal" drift-dissipative instability,[3] which occurs because the velocity of drift waves is below v^*. According to Refs. 5 and 6, vortices can also be amplified. Since the density is frozen in the plasma according to (1), we would expect an increase in $|\nabla\phi|$, i.e., an increase in the plasma rotation velocity in a vortex. The displacement velocity u and the shape of the vortex should vary slowly.

The numerical calculations show that the maximum vortex potential indeed initially increases and then reaches saturation. The maximum density falls off slightly in the process (Fig. 1). The vortex energy, (3), however, continues to grow, because of the increase in the potential gradient and thus the particle rotation velocity in the

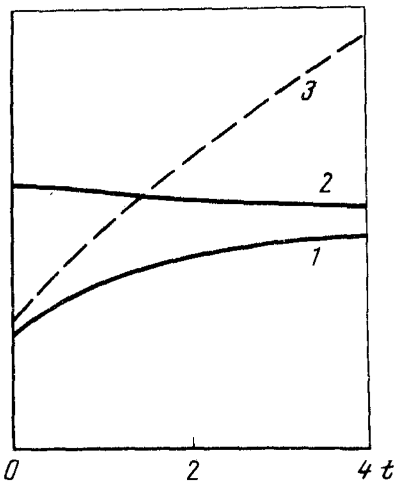

FIG. 1. 1,2—Time evolution of the maximum potential ϕ (curve 1) and of the quantity $(T_i/-e)n_{max}$, where n_{max} is the maximum value of the density perturbation (curve 2) in a vortex; 3—vortex energy W. The properties are plotted in arbitrary units; the time is expressed in units of $(a\ |u|)^{-1}$. The model value of the viscosity is $(a_\mu/|u|) = 0.2$, $u = v^*/2$.

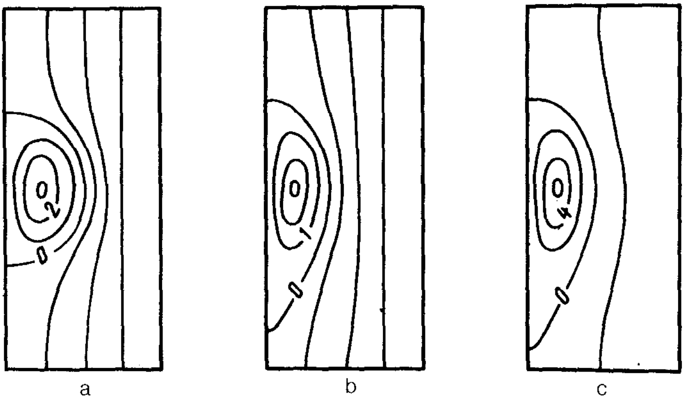

FIG. 2. Contour map of the dimensionless perturbations of the potential, $\hat{\phi} = ac\phi/uB_0 - ax$, and the density, $\hat{n} = (a/k)(n - kx)$, in a vortex. a: $t = 0$; the contour lines of $\hat{\phi}$ and $\hat{n}$ coincide; the lines of the levels 3, 2, 1,..., are shown. b: $t = 4$; contour lines are shown for $\hat{n} = 3, 2, 1,...$. c: $t = 4$; contour lines are shown for $\hat{\phi} = 6.4, 2,...$. The left sides of the figures, which have been omitted, are antisymmetric with respect to the right sides.

vortex (Fig. 2). The vortex remains topologically stable, although it deviates significantly from the steady-state solution for the case without viscosity.[2,4]

In summary, this study has shown that in the stability region of flute waves dipole vortices are topologically stable and are amplified by a viscosity. We also note that the possibility of a dissipative amplification of drift-Alfvén vortices was mentioned in Refs. 5 and 6, but as yet we have no stability analysis or numerical simulation for that process. The results of the present study, in view of the similarity of the equations, are indirect evidence in favor of the conclusions of Refs. 5 and 6.

[1]B. B. Kadomtsev, in: Reviews of Plasma Physics, Vol. 2, Consultants Bureau, New York, 1966.
[2]V. P. Pavlenko and V. I. Petviashvili, Fiz. Plazmy **9**, 1034 (1983) [Sov. J. Plasma Phys. **9**, 603 (1983)].
[3]A. B. Mikhailovskii, Theory of Plasma Instabilities, Vol. 2, Consultants Bureau, New York, 1974.
[4]V. D. Larichev and G. M Reznik, Dokl. Akad. Nauk SSSR **231**, 1077 (1976) [Sov. Phys. Dokl. **21**, 581 (1976)].
[5]V. I. Petviashvili and O. A. Pokhotelov, Fiz. Plazmy **12**, 1127 (1986) [Sov. J. Plasma Phys. **12**, 651 (1986)].
[6]V. I. Petviashvili and I. O. Pogutse, Pis'ma Zh. Eksp. Teor. Fiz. **43** 268 (1986) [JETP Lett. **43**, 343 (1986)].

Translated by Dave Parsons