Convective heat transfer and other thermoelectric effects in high-temperature superconductors

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The convection mechanism for heat transfer may be significant and observable in high-temperature superconductors. Some other questions, related to thermoelectric effects in high-temperature superconductors, are also discussed.

The normal current density $\mathbf{j}_n = b_n \vee \nabla T$ (here and below, we are ignoring signs) is nonzero in homogeneous and isotropic superconductors with a temperature gradient, but is it canceled by the superconducting current $\mathbf{j}_s$, so the total current density vanishes: $\mathbf{j} = \mathbf{j}_s + \mathbf{j}_n = 0$. Even in this case, however, the existence of the current $\mathbf{j}_n$ gives rise to an additional, convective heat transfer. In inhomogeneous and also anisotropic superconductors, we would generally have $\mathbf{j} \neq 0$, in contrast, and certain other thermoelectric effects could occur along with convective heat transfer. This circumstance was pointed out a very long time ago,[1] and the situation has been discussed in reviews,[2,3] but thermoelectric effects in the superconducting state have nevertheless attracted little interest and are frequently ignored altogether. There are some rich possibilities in this field,[2,3] particularly for high-temperature superconductors. We would like to emphasize the latter point here.

The heat flux in a superconductor is described by $\mathbf{q} = \kappa \widehat{\nabla} T$, where $\kappa = \kappa_{lt} + \kappa_{el} + \kappa_c$ and κ_{lt}, κ_{el}, and κ_c correspond to the components from the lattice, normal electrons, and convection, respectively. Under conditions such that the Wiede-

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mann-Franz law holds we have $\kappa_{el} = (\pi^2 k_B^2/3e^2)T\sigma_n$ where $\sigma_n = j_n/E$ is the electrical conductivity of the normal electrons. The convection flux $\mathbf{q}_c = \kappa_c \widehat{\nabla} T$ stems primarily from a rupture of superconducting pairs at a higher temperature T_2 and from the formation of pairs from normal electrons at $T_1 < T_2$ where the current $\mathbf{j}_n$ converts into a current $-\mathbf{j}_s$. We thus have $\mathbf{q}_c \sim j_n \Delta/e$ and $\kappa_c \sim b_n \Delta/e$ where 2Δ is the formation energy of a pair with a charge $2e$. Also using the Wiedemann-Franz law, we find

$$\kappa_c/\kappa_{el} \sim 3eS\Delta/\pi^2 k_B^2 T, \tag{1}$$

where $S \equiv d\mathscr{E}/dt = b_n/\sigma_n$ is the Seebeck coefficient or, as it is also known, the differential thermal emf. For free electrons we would have $S = \pi^2 k_B^2 T/3eE_F$ (Ref. 2, for example), where E_F is the Fermi energy. By virtue of (1) we then have

$$\kappa_c/\kappa_{el} \sim \frac{\Delta(T)}{E_F} \sim \frac{k_B T_c}{E_F}, \tag{2}$$

where we have set $\Delta(T) \sim k_B T_c$ in deriving the last expression. Estimates of this sort are meaningful for ordinary superconductors in a certain region below the critical temperature T_c and indeed are the estimates which are usually made.[4–6] For ordinary superconductors with $T_c \sim 1$–10 K and $E_F \sim 3$–10 eV we would have $\kappa_c/\kappa_{el} \lesssim 10^{-4}$, according to (2), so the convective heat transfer is assumed to be negligible.[2,4–6]

For high-temperature superconductors the situation may be different. According to the estimates of Ref. 7, for example, the Fermi energy in the high-temperature superconductors is $E_F \sim 0.3$ eV; hence we would have $\kappa_c/\kappa_{el} \sim k_B T_c/E_F \sim 3\times 10^{-2}$ (at $T_c \sim 10^2$ K). With $S \sim 10^{-5}$ V/C $\sim 3\times 10^{-8}$ (cgs units)/K (more on this below) and $\Delta/kT \sim 1$, estimate (1) also gives us $\kappa_c/\kappa_{el} \sim 3\times 10^{-2}$. Estimates (1) and, especially (2) are so crude—they ignore the anisotropy and possibly complex nature of the Fermi surface—that situations in which the relation $\kappa_c/\kappa_{el} \gtrsim 1$ holds for high-temperature superconductors are totally conceivable. What would be the consequences? At sufficiently large values of κ_c, the measured thermal conductivity κ should evidently begin to increase with decreasing $T < T_c$. Later on, as the normal electrons are "frozen out," κ_c, like κ_{el}, would decrease and approach zero in the limit $T \to 0$. This, however, is precisely what is observed[8]; the maximum of κ increases with increasing value of the component κ_{el} and thus of κ_c. At the moment, we are by no means in a position to assert that a convective heat transfer is being observed, since this increase in κ could be explained by an increase in κ_{lt} due to a decrease in the scattering of phonons by the normal electrons which are frozen out as T_c is lowered. Furthermore, an increase in κ at $T < T_c$ has been observed[5] in certain low-temperature alloys. A resolution of this question will require experimental and theoretical research on the thermal conductivity and thermoelectric effects in superconductors.

Above T_c, the values of S and σ and thus b can be measured directly. One might suggest that just below T_c we would have values $b_n \approx b(T_c)$ and $\sigma_n \approx \sigma(T_c)$. According to Ref. 9, the coefficient b_n increases by several orders of magnitude during anisotropic superconducting pairing. In general, the situation is not clear. If we use the values of $S(T_c)$ and $\sigma(T_c)$, on the other hand, then for high-temperature superconductors with, say, $S \sim 10^{-5}$ V/K and $\sigma = 1/\rho \sim 10^3$ S/cm, we would have $b \sim 10^{-2}$ V/

($\Omega \cdot$cm$\cdot$K). At the same time, for Sn, for example, we would have[2] $\sigma \sim 10^9$ S/cm, $S \sim 10^{-7}$ V/K, and $b \sim 10^2$ V/($\Omega \cdot$cm$\cdot$K). Both the magnetic flux which arises in the nonuniform thermoelectric circuit and the magnetic flux in an anisotropic, nonuniformly heated superconducting sample, are proportional to $b_n \delta^2$, where δ is the field penetration depth.[2] Accordingly, if b_n in ceramic high-temperature superconductors is small in comparison to that in ordinary superconductors, the corresponding effects will be weakened. However, we do not know just what the values of b_n are for single crystals of high-temperature superconductors. We might add that there is a clear need for corresponding experiments[2,3] on ceramics also, particularly in the case of quasi-closed (toroidal) configurations[3] (see also Refs. 10 and 11 in this connection). In summary, we can only stress that research on thermoelectric effects (including convective heat transfer) in the superconducting state has some distinctive features in the case of high-temperature superconductors and is not of minor interest (see also Ref. 12 in this connection).

[1]V. L. Ginzburg, Zh. Eksp. Teor. Fiz. **14**, 177 (1944); J. Phys. USSR **8**, 148 (1944).
[2]V. L. Ginzburg and G. F. Zharkov, Usp. Fiz. Nauk **125**, 19 (1978) [Sov. Phys. Usp. **21**, 381 (1978)].
[3]D. J. Van Harlinger, Physica **B109–110**, 1710 (1982).
[4]P. G. Klemens, Proc. Phys. Soc. **A66**, 576 (1953).
[5]P. G. Klemens, Handb. Phys. **14**, 198 (1956).
[6]B. T. Geĭlikman and V. Z. Kresin, *Kinetic and Time-Varying Phenomena in Superconductors*, Nauka, Moscow, 1972.
[7]L. P. Gor'kov and N. B. Kopnin, Usp. Fiz. Nauk **156**, 117 (1988) [Sov. Phys. Usp xx, xxx (1988)].
[8]A. Jezowski, A. J. Zaleski, M. Ciszek *et al.*, Helv. Phys. Acta **61**, 438 (1988).
[9]C. J. Pethick and D. Pines, Preprint, 1987, 1988.
[10]V. L. Ginzburg, G. F. Zharkov, and A. A. Sobyanin, Low Temp. Phys. **56**, 195 (1984).
[11]G. F. Zharkov, in *Superconductivity, Superdiamagnetism, Superfluidity* (ed. V. L. Ginzburg), Mir, Moscow, 1987, p. 126.
[12]V. L. Ginzburg, in *Proceedings of the Eighteenth Conference on Low Temperature Physics*, Part 3, 2046, 1987.

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