Interaction of neutrinos with collective oscillations of neutron matter

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The interaction of neutrinos with collective oscillations in the form of spin waves and zero sound in a neutron Fermi liquid, whose quanta are bosons, contributes substantially to the relaxation of neutrinos in the late stage of gravitational collapse and during the cooling of the neutron residue of a star.

In the late stage of the gravitational collapse of a star, and during the cooling of its neutron residue, with a neutron density reaching values $n_0 \gtrsim 10^{38}$ cm^{-3}, collective oscillations in the form of spin waves and zero sound, for which the quanta are bosons, can exist in the neutron Fermi liquid. The relaxation of neutrinos through the exchange of energy with these collective quanta may compete with the scattering of neutrinos by electrons since the exchange of energy between the neutrinos and the degenerate electrons of the medium is largely suppressed by the degeneracy of the neutrino gas.

In this analysis of the scattering of neutrinos by fluctuations of the particle density and of the spin density in the neutron matter, we start from the interaction of neutrinos with the neutrons of the medium through neutral currents in the point four-Fermion approximation.[1] This approximation is valid if the energy of the neutrinos is much smaller than the mass of the Z boson.[1)] Assuming that the neutrons are nonrelativistic, we can write the νN interaction in the form

$$\hat{\mathcal{H}} = \frac{G_F}{\sqrt{2}}\left[\left(\bar{\Psi}_\nu \frac{1+\gamma_5}{2} \gamma^0 \Psi_\nu\right)(\varphi^+\varphi) - g_A\left(\bar{\Psi}_\nu \frac{1+\gamma_5}{2}\vec{\gamma}\Psi_\nu\right)(\varphi^+\hat{\vec{\sigma}}\varphi)\right], \tag{1}$$

where $G_F = 10^{-5}/m_p^2$ (m_p is the mass of a proton), $g_A = 1.25$ is the axial constant of the weak interaction, the operators Ψ_ν and φ represent the free neutrino field and the free neutron field, and $\hat{\sigma}$ are the Pauli matrices. We assume that the bispinor amplitudes of the neutrinos are normalized by the condition $\bar{\nu}_q \nu_q = 2\epsilon$, $q = (\epsilon = |\mathbf{q}|, \mathbf{q})$, and we assume that the spinor amplitudes of the neutrons are normalized by $u_\mathbf{p}^+ u_\mathbf{p} = 1$.

Taking the expectation value of interaction (1) over a volume of the medium which physically is exponentially small, and using the nonequilibrium one-particle neutron density matrix $f_{ss'}(\mathbf{p},\mathbf{r},t)$, we find

$$\hat{\mathcal{H}} = \frac{G_F}{\sqrt{2}}\left[\left(\bar{\Psi}_\nu \frac{1+\gamma_5}{2} \gamma^0 \Psi_\nu\right) n(\mathbf{r},t) - g_A\left(\bar{\Psi}_\nu \frac{1+\gamma_5}{2}\vec{\gamma}\Psi_\nu\right)\delta\vec{\sigma}(\mathbf{r},t)\right], \tag{2}$$

where $n(\mathbf{r},t) = n_0 + \delta n(\mathbf{r},t)$, and n_0 is the equilibrium density of neutrons. Small perturbations of the neutron density, $\delta n(\mathbf{r},t)$, and of the spin density, $\delta\sigma(\mathbf{r},t)$, are independent in the linear approximation.[2] The square of the matrix element for the transition of neutrinos from state q to state q' due to interaction (2) is therefore

$$\overline{|M_{qq'}|^2} = \frac{G_F^2}{2}\mathrm{Sp}\left[\frac{1+\gamma_5}{2}\gamma^0(\gamma q')\gamma^0(\gamma q)\right]\langle\delta n^2\rangle_{\omega,\mathbf{k}}$$

$$+g_A^2\frac{G_F^2}{2}\mathrm{Tr}\left[\frac{1+\gamma_5}{2}\gamma_i(\gamma q')\gamma_j(\gamma q)\right]\langle\delta\sigma_i\delta\sigma_j\rangle_{\omega,\mathbf{k}}, \tag{3}$$

where $\omega=\epsilon-\epsilon'$; $\mathbf{k}=\mathbf{q}-\mathbf{q}'$; and $\langle\delta n^2\rangle_{\omega,\mathbf{k}}$ and $\langle\delta\sigma_i\delta\sigma_j\rangle_{\omega,\mathbf{k}}$ are the spectral distributions of the fluctuations in the particle density and the spin density, respectively, averaged over a Gibbs distribution.

The neutrons of the medium are assumed to be degenerate; i.e., we assume $T\ll\epsilon_F$ (T is the temperature of the medium, $\epsilon_F=p_F^2/2m^*$ is the Fermi energy, and m^* is the mass of a neutron quasiparticle). The interaction between quasiparticles at a small momentum transfer, $\delta p\ll P_F$, $\delta E\ll E_F$, near the Fermi surface can be written in the s-scattering approximation[3]:

$$\hat{\mathcal{F}}(\mathbf{p},\mathbf{p}')=\frac{\pi^2}{p_F m^*}(F_0+G_0\hat{\vec\sigma}\hat{\vec\sigma}'), \tag{4}$$

where the dimensionless constants $F_0\sim G_0\sim 1$ must be found by comparing the theory with experiment. The isotopic level shift and the quadrupole moments of the nuclei are explained satisfactorily with $F_0=1.35$; data on the magnetic moments of nuclei yield[3] $G_0=0.5$.

According to the theory of a Fermi liquid,[4] under the condition $\omega\tau\gg 1$ ($\tau\sim T^{-2}$ is the time scale between collisions of the quasiparticles) the spectral distribution of the density fluctuations $\langle\delta n^2\rangle_{\omega,\mathbf{k}}$ has sharp maxima at frequencies corresponding to zero sound, whose velocity u_0 is determined by the equation

$$\frac{1}{F_0}=\frac{u_0}{2v_F}\ln\frac{u_0+v_F}{u_0-v_F}-1. \tag{5}$$

Here $v_F\ll 1$ is the velocity of the quasiparticles at the Fermi surface. Correspondingly, the spectrum of the spin-density fluctuations $\langle\delta\sigma_i\delta\sigma_j\rangle_{\omega,\mathbf{k}}$ has sharp maxima at frequencies which correspond to spin waves, whose velocity u_s is found from an equation which differs from (5) only in that F_0 is replaced by G_0, and u_0 by u_s. Using the values given above for F_0 and G_0, we find $u_0/v_F=1.08$ and $u_s/v_F=1.005$. Near the maxima, the spectra of the fluctuations are[5]

$$\langle\delta n^2\rangle_{\omega,\mathbf{k}}=\frac{2m^*p_F}{\pi}A\frac{\mathrm{sign}\,\omega}{[1-\exp(-\omega/T)]}ku_0\delta(|\omega|-ku_0), \tag{6}$$

$$\langle\delta\sigma_i\delta\sigma_j\rangle_{\omega,\mathbf{k}}=\delta_{ij}\frac{2m^*p_F}{\pi}B\frac{\mathrm{sign}\,\omega}{[1-\exp(-\omega/T)]}ku_s\delta(|\omega|-ku_s), \tag{7}$$

where

$$A = (u_0^2 - v_F^2)/F_0\,[(1+F_0)v_F^2 - u_0^2] \approx 0.1\,,$$

$$B = (u_s^2 - v_F^2)/G_0[(1+G_0)v_F^2 - u_s^2] \approx 0.03.$$

Using (3), (6) and (7), we find the probability for the scattering of a neutrino from the state $\mathbf{q}$ to the state $\mathbf{q}' = \mathbf{q} - \mathbf{k}$. For scattering by zero sound we have

$$dw_0 = \frac{A}{\pi} G_F^2 m^* p_F \left(1 + \frac{\mathbf{q}\mathbf{q}'}{\epsilon_{\mathbf{q}}\epsilon_{\mathbf{q}'}}\right) k u_0 [(N_0+1)\delta(\epsilon_{\mathbf{q}} - \epsilon_{\mathbf{q}'} - ku_0) + N_0 \delta(\epsilon_{\mathbf{q}} - \epsilon_{\mathbf{q}'} + ku_0)] \frac{d^3k}{(2\pi)^3} \tag{8}$$

For scattering by spin waves we have

$$dw_s = \frac{B}{\pi} g_A^2 G_F^2 m^* p_F \left(3 - \frac{\mathbf{q}\mathbf{q}'}{\epsilon_{\mathbf{q}}\epsilon_{\mathbf{q}'}}\right) k u_s [(N_s+1)\delta(\epsilon_{\mathbf{q}} - \epsilon_{\mathbf{q}'} - ku_s) + N_s \delta(\epsilon_{\mathbf{q}} - \epsilon_{\mathbf{q}'} + ku_s)] \frac{d^3k}{(2\pi)^3}\,, \tag{9}$$

where $N_{0,s} = [\exp(ku_{0,s}/T) - 1]^{-1}$ is the number of quanta of frequency $ku_{0,s}$ at thermodynamic equilibrium. The possible values of the momentum transfer during the scattering are determined by the energy conservation law $\epsilon_{\mathbf{q}} - \epsilon_{\mathbf{q}'} = \pm ku_{0,s}$, from which we find $k \leqslant 2\epsilon_{\mathbf{q}}(1 \mp u_{0,s})$.

Figures 1 and 2 show two neutrino relaxation times calculated from Eqs. (8) and (9): $\tau_1 \equiv -\epsilon/\langle\epsilon' - \epsilon\rangle$ and $\tau_2 \equiv \epsilon^2/\langle(\epsilon' - \epsilon)^2\rangle$. These times were determined with the help of the first and second moments of the energy transfer:

$$\langle(\epsilon' - \epsilon)^n\rangle \equiv \int dw_{0,s}(\epsilon_{q'} - \epsilon_q)^n [1 - f_\nu(\epsilon_{q'})], \tag{10}$$

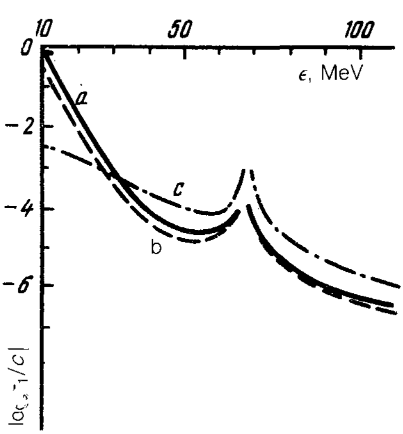

FIG. 1. Relaxation time of the neutrino energy, $\tau_1(\epsilon)$. a—For the scattering of neutrinos by zero sound; b—for scattering by spin waves; c—for scattering by electrons. These calculations were carried out for $\mu_e = 80$ MeV, $\mu_\nu = 63$ MeV, $T = 5$ MeV, and $n_0 = 10^{38}$ cm^{-3}.

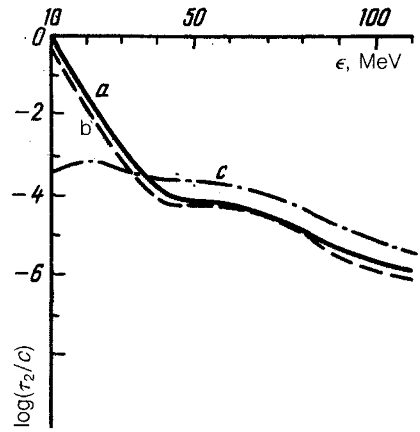

FIG. 2. Time scale of the diffusion of neutrinos along the energy scale, $\tau_2(\epsilon)$. a—For the scattering of neutrinos by zero sound; b—for scattering by spin waves; c—for scattering by electrons ($\mu_e = 80$ MeV, $\mu_\nu = 63$ MeV, $T = 5$ MeV, and $n_0 = 10^{38}$ cm^{-3}).

where $f_\nu(\epsilon) = [1 + \exp(\epsilon - \mu_\nu)/T]^{-1}$ is the distribution function of the equilibrium neutrinos, with a chemical potential μ_ν. According to definition (10), $\tau_1(\epsilon)$ is the time scale of the energy exchange between the neutrinos and the collective oscillations of the neutron Fermi liquid, while $\tau_2(\epsilon)$ is the time scale of the diffusion of neutrinos along the energy scale.[6] Shown for comparison in Figs. 1 and 2 are curves of $\tau_{1,2}(\epsilon)$ for the scattering of neutrinos by electrons of the medium, with a chemical potential μ_e. [See Ref. 7 regarding the details of the calculations of $dw_e(\mathbf{q},\mathbf{q}')$ for the scattering of neutrinos by electrons.]

The degeneracy of the neutrino gas significantly hinders an exchange of energy between the neutrinos and the degenerate electrons of the medium, but it does not prevent an exchange of energy with the quanta of collective oscillations of the neutron Fermi liquid, which are bosons.

As a result, in the late stage of the collapse of a star, and during the cooling of the its neutron residue, the scattering of neutrinos by zero sound and by spin waves competes with the scattering by electrons in the neutrino thermalization process.

[1)]We are using a system of units with $\hbar = c = 1$, the Feynman metric $q_\mu q^\mu = \epsilon^2 - \mathbf{q}^2$, and the standard representation of the Dirac γ matrices, with $\gamma_5^{\dagger} = \gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$.

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