The Josephson effect in granular superconductors

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The present model of a granular superconductor with a scatter of the critical currents, δI_c, is described by equations like those that are used for a Josephson junction. The I–V characteristics are obtained, with allowance for δI_c. The Shapiro steps are found to be present in a large strip, but one of finite width, and magnetooscillations are found to occur in an infinite strip.

It has been established that a high-T_c superconducting ceramic is a granular system with Josephson junctions between the grains (see, e.g., Ref. 1). Ceramics are known to exhibit Josephson effects, i.e., a coherent response to the application of the magnetic field H and an alternating signal. Both effects have been observed when the junction size is much greater than not only ξ and λ but also the grain size (see, e.g., Refs. 2–4). Some investigators have assumed that one or more junctions might operate. Such an assumption is plausible only near T_c, where the fluctuations suppress the superconductivity in the Josephson junctions and the percolation effects are appreciable. Far from T_c, where these effects have actually been observed, the current flows through many Josephson junctions and the validity of this assumption is questionable.

In the present letter we consider a model of a granular superconductor with Josephson junctions with various critical currents I_c. The system which we are considering is described by the same equations as those used for inhomogeneous Josephson tunnel junctions. Typical size of a junction, in which coherent effects can be observed, may be, as will be shown below, quite large. We will also determine the current-voltage characteristic, with allowance for the scatter in I_c.

Let us assume that grains form a square lattice with Josephson junctions situated at the points at which the grains come in contact. As usual, we represent the phase of the grain as $\tilde{\chi}(\mathbf{r}_i) = \chi(\mathbf{r}_i) - (2\pi/\Phi_0)\int_{\mathbf{r}_i}^{\mathbf{r}} d\mathbf{r}'\mathbf{A}(\mathbf{r}')$, where $\mathbf{r}_i$ is the coordinate of the grain center. Let us consider two types of contours with circulating currents I_1 and I_2 (Fig. 1). The currents I_1 are related to the currents flowing through the Josephson junctions, I_J, each of which is characterized by the phase difference $\varphi(\mathbf{r}_i) = \tilde{\chi}(\mathbf{r}_i + a) - \tilde{\chi}(\mathbf{r}_i)$, where $\mathbf{a} = a_\alpha$, $\alpha = (x,y)$ (Fig. 1). Analyzing the continuum limit, where $\varphi(\mathbf{r})$ remains essentially constant on the dimension $\sim a$, we find

$$\mathrm{curl}\,\mathbf{I}_1 = \mathbf{I}_J, \tag{1}$$

where $\mathbf{I}_1 = (0,0,I_1)$, $I_{J\alpha} = I_c[(C\hbar/2eI_c)\partial_{tt}^2\varphi_\alpha + \tau_c\partial_t\varphi_\alpha + \sin\varphi_\alpha]$, $\tau_c^{-1} = 2eI_cR/\hbar$, and $\varphi_\alpha = a[\widehat{\nabla}_\alpha\chi - 2\pi A_\alpha/\Phi_0]$; Φ_0 is a fluxoid. We integrate the expression for the current density in each grain $j_\alpha = \beta[\widehat{\nabla}_\alpha\chi - 2\pi A_\alpha/\Phi_0]$ over the contours 1 and 2, assuming, for simplicity, that $a \gg \lambda$ (such an approach, which is used in the analysis of SQUIDs, was applied to granular systems in Refs. 5). For a magnetic flux through

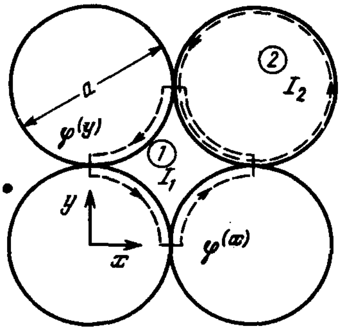

FIG. 1. Model of a granular superconductor. The dashed lines represent the contours of integration.

contour 1 we have

$$\Phi_1 = \bar{H}S = \Phi_1^{(ex)} + L_1 I_1 - S^{-1} \int d^2 r'_\alpha [L_{11}(r_\alpha - r'_\alpha) I_1(r'_\alpha) + L_{12}(r_\alpha - r'_\alpha) I_2]. \qquad (2)$$

Here $\overline{H}' = \text{curl } A$, $\Phi_1^{(ex)}$ is the external magnetic field flux, L_1 is the self-inductance, and $L_{11(12)}$ are the coefficients of mutual inductance of contours 1 and 1(2).

If $H^{(ex)}$ is smaller than H_{c1} for a given grain, the flux in contour 2 is zero:

$$\Phi_2 = L_2 I_2 - S^{-1} \int d^2 r'_\alpha [L_{22}(r_\alpha - r'_\alpha) I_2(r'_\alpha) + L_{21}(r_\alpha - r'_\alpha) I_1(r'_\alpha)] = 0. \qquad (3)$$

If the transverse dimensions of a system, $l_{x,y}$, are much larger than the longitudinal dimension (in H), l_z, retention of the flux leads to a coupling between $L_{1(2)}$ and $L_{12(21)}$:

$$L_{1(2)} = S^{-1} \int d^2 r_\alpha [L_{11(22)}(r_\alpha) + L_{21(12)}(r_\alpha)]. \qquad (4)$$

Equations (1)–(4) describe this system.

Let us consider, for example, a lattice vortex for a simple case in which L_{11} and L_{12} in Eq. (2) can be ignored relative to L_1. Far from the vortex center we can then write the standard equation for the Fourier components $\mathbf{A}(\mathbf{q})$

$$(l_0^2 q^2 + 1) A_\alpha(q) = i q_\alpha (\chi/2\pi) \Phi_0, \qquad (5)$$

where a typical vortex, $l_0 = (S\Phi_0/2\pi L_1 I_c)^{1/2} \sim (\Phi_0 c/j_c a)^{1/2}$, may be quite large[5,6]; specifically, it may be larger than the grain size a (for $j_c \sim 10^3$ A/cm^2 and $a \sim 1\ \mu$m we have $l_0 \sim 10\ \mu$m). If $l_z \ll l_{x,y}$, we must take the coefficients $L_{11(12)}$ and Eq. (4) into account. We will then obtain Eq. (5), in which $l_0^2 q^2$ should be replaced by $l_0^2 q^2/F(q)$, where the function $F(q) \sim c_1 q l_z$ when $q \ll l_z^{-1}$, and $c_1 \sim 1$. The decay of H from the vortex center in this case obeys the power law.

Let us analyze the Josephson effects. We will first consider a simple case, $l_z \gg l_{x,y}$. The critical current is $I_c(r_\alpha) = I_c[1 + f(r_\alpha)]$, where $\langle f(\mathbf{r}_\alpha) f(\mathbf{r}'_\alpha)\rangle = S f_0^2 \delta(\mathbf{r}_\alpha - \mathbf{r}'_\alpha)$, and $f_0^2 = \langle \Delta I_c^2 \rangle / I_c^2$. We will ignore the fluctuations of other quantities (such as R) (this is completely justifiable in the case of SNS-type Josephson junctions). For the phase difference $\varphi = 2\pi a A_y/\Phi_0$ we then find

$$l_0^2\nabla^2\varphi-\tau_c\partial_t\varphi=(1+f)\sin\varphi. \qquad (6)$$

With the boundary conditions for a band of width l_x (the current flows along the y axis) we have

$$(\partial\varphi/\partial x)_{x=\pm(l_x/2)}=(H\pm 4\pi I/cl_z)(2\pi a/\Phi_0). \qquad (7)$$

Here we have ignored the capacitive current which causes plasma vibrations, assuming the frequencies to be not particularly large, $\omega RC\ll 1$. Equation (6) also describes the inhomogeneous Josephson tunnel junction or a commensurate charge density wave which interacts with the impurities.[7] If the fluctuations are small ($f_0\ll 1$), then the fluctuations φ will also be small in the static case.[7] In the dynamic case ($I>I_c$), however, arbitrarily small fluctuations f cause the Shapiro steps, ΔI_S, to disappear if the system is sufficiently large.

Let us determine the I–V characteristic and ΔI_S under the assumption that $l_x\ll l_0$ and that the current is large: $I\gg I_c$. In the absence of H the solution of (6) can be sought in the form $\varphi=\omega t+\theta(r_\alpha)+\psi(r_\alpha,t)+I(t)\dfrac{x^2}{2I_c l_0^2}$, where $I(t)=I+I_\omega\sin(\omega t)$, the function $\psi(r_\alpha,t)$ is small ($\psi\ll 1$) and oscillates with time, and the function $\theta(r_\alpha)$ is comprised of a smoothly varying part, $\theta_1(r_\alpha)\gtrsim 1$, and a small part, $\theta_2\ll 1$, which varies markedly over the lengths $\lesssim l_0\ (I_c/I)^{1/2}$. We substitute $\varphi(r_\alpha,t)$ in (6) and expand $\sin\varphi$ in a series in powers of ψ, x^2, and θ_2 to the first order in ψ and to the zeroth order in $x^2\sim l_x^2$ and θ_2. Having found ψ, we can determine the I–V characteristic for $I\gg I_c$.

$$\delta I/I_c=I/I_c-[\tau_c\omega+(2\omega\tau_c)^{-1}]=(f_0^2/2)\sum_q\frac{\omega\tau_c S/l_0^2}{(\omega\tau_c)^2+(ql_0)^4}=\frac{\pi f_0^2 S/l_0^2}{2\sqrt{2\omega\tau_c}} \qquad (8)$$

The frequency ω is a consequence of the voltage on a segment of length l_y: $2eV=\hbar\omega l_y/a=\hbar\omega N$, where N is the number of grains along a segment of length l_y. The right side of (8) gives the correction to the I–V characteristic due to the scatter of I_c and at large values of the current determines the manner in which the I–V characteristic approaches the resistance curve. For $\theta_1(q)$ we have $\theta_1(q)=-(\omega\tau_c f(q)/(l_0^2q^2))((\omega\tau_c)^2+l_0^4q^4)^{-1}$; i.e., in the case of small values of q the quantity θ_1 diverges, causing ΔI_S to be suppressed. For a Shapiro step we find

$$\Delta I_S/I_c=(I_\omega/I)\langle\cos\theta_1\rangle=(I_\omega/I)\exp[-\langle\theta_1^2(r_\alpha)\rangle/2]\sim\exp[-(f_0/l_0)^2S(l_y/l_0)^3(I_c/I)^2].$$

If the bridge is long $[l_y^3>l_S^3=l_0^5(I/I_c)^2/(f_0^2S),]$ for example, ΔI_S is small.

In the case of samples with $l_{x,y}\gg l_z$ (films, for example) the results remain qualitatively the same but change quantitatively, since the $\theta_1(q)$ and $\psi(q)$ dependences change because of the appearance of the function $F(q)$ in (6) [see the discussion of Eq. (5)]. In particular, the fluctuation-related change in the I–V characteristic is described by $\delta I/I_c=f_0^2\pi(l_z/l_0)\ \mathrm{sgn}\ V$, rather than by (7); i.e., an excess current which does not depend on $|V|$ and which is caused by the I_c fluctuations, rather than by the Andreev reflection,[8] is produced. The criterion for coherence also changes: ΔI_S is not small if $l_y<l_S=l_0(I/I_c)^2(l_0/l_z)^2(Sf_0^2/l_0^2)^{-1}$.

In the presence of a field H a similar procedure may be used to determine the I–V characteristic. It turns out, however, that the role of fluctuations changes. If we once again assume that $l_x < l_0$, the oscillations of the I–V characteristic $\delta I/I_c = (I_c/2I)(\cos\widetilde{\Phi})/\widetilde{\Phi}^2$ (where $\widetilde{\Phi} = 2\pi H l_x a/\Phi_0$ and $I \gg I_c$) will occur regardless of the length l_y of the bridge. Accordingly, a bridge in a high-T_c superconductor may exhibit magnetooscillations[3] if l_x is not particularly large ($l_x \lesssim l_0$) and l_y is of arbitrary size. The Shapiro steps, on the other hand, are seen only in a bridge of finite length, $l_y < l_S$.

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