# Critical current of the Josephson junctions with randomly distributed vortices

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The magnetic-field dependence of the critical current of the Josephson junction is determined for various concentrations of the Abrikosov vortices in the superconducting junction edges.

The Josephson junctions whose edges are made of "hard" type-II superconductors (Nb, NbN, $Nb_3Sn$) have recently been studied extensively.[1,2] In an external magnetic field such superconductors give rise to Abrikosov vortices which are pinned on various defects. Because Abrikosov vortices have a nonuniform magnetic field, the dependence of the critical current on the external magnetic field may differ considerably from the ordinary Fraunhofer dependence characteristic of homogeneous Josephson junctions.[3] Let us determine the critical current of a Josephson junction, whose

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edges have randomly distributed Abrikosov vortices with the axes running parallel to the plane of the junction.

Let us assume that the Josephson junction is situated in an external magnetic field $H$ which is parallel to the plane of the junction. The critical current is given by the equation[3]

$$I_c^2 = j_0^2 \left| \int_0^L \exp[i\varphi(x)]dx \right|^2, \tag{1}$$

where $j_0$ is the critical current density, and $L$ is the length of the junction. The difference between the phases $\varphi$ depends on the external field and on the coordinates $x_i$, $y_i$ of the Abrikosov vortices at the junction's edges[4]

$$\varphi = \sum_{i=1}^{N} -\frac{2}{\lambda}\int_0^x Z(x-x_i, y_i)dx + \frac{2\pi\phi x}{\phi_0 L},$$

$$Z(x-x_i, y_i) = |y_i|[(x-x_i)^2+y_i^2]^{-1/2} K_1\left\{\frac{[(x-x_i)^2+y_i^2]^{1/2}}{\lambda}\right\}, \tag{2}$$

where $\phi = 2HL\lambda$, $\lambda$ is the London penetration depth of a superconductor, and $K_1(x)$ is a modified Bessel function.

To determine the average current flowing through the junction, $\overline{I_c^2}$, we will average expression (1) over various positions of the Abrikosov vortices. Assuming that they are distributed randomly in the plane of the junction, we find, just as in Ref. 5, the following expression ($S$ is the area of the junction):

$$\overline{I_c^2} = j_0^2 \int_0^L dx_1 \int_0^L dx_2 \exp[2\pi i\phi(x_1-x_2)/\phi_0 L]$$

$$\times \left\{\int \frac{dxdy}{S} \exp\left[-i\frac{2}{\lambda}\int_{x_2}^{x_1} Z(t-x,y)dt\right]\right\}^N = j_0^2 \int_0^L dx_1 \int_0^L dx_2$$

$$\times \exp\left\{\frac{2\pi i\phi(x_1-x_2)}{\phi_0 L} + n\int dxdy \left\{\exp\left[-i\frac{2}{\lambda}\int_{x_2}^{x_1} Z(t-x,y)dt\right] - 1\right\}\right\}, \tag{3}$$

where $n$ is the concentration of the Abrikosov vortices at the superconducting edges.

If the concentration of Abrikosov vortices is small ($n\lambda^2 \ll 1$), we can easily calculate the integrals in (3) (the principal contribution comes from $|x_1 - x_2| \gg \lambda$) and we obtain

$$\overline{I_c^2} = j_0^2 \int_0^L dx_1 \int_0^L dx_2 \exp\left\{2\pi i\left(\frac{\phi}{\phi_0 L} - \alpha_2 n\lambda\right)(x_1-x_2) - \alpha_1 n\lambda|x_1-x_2|\right\}$$

$$\alpha_1 = 2\lambda^{-1}\int_0^\infty dy\{1-\cos(2\pi e^{-y/\lambda})\} \approx 2\ln 2\pi;$$

$$\alpha_2 = (\pi\lambda)^{-1}\int_0^\infty dy \sin 2\pi e^{-y/\lambda} \approx 1/2. \tag{4}$$

To obtain Eq. (4), we made use of the fact that the function $K_1(t)$ decays exponentially when $t \gg 1$ and we also used the integral[4]

$$\frac{2}{\lambda}\int_{-\infty}^{\infty} |y|(t^2+y^2)^{-1/2} K_1\left[\frac{(t^2+y^2)^{1/2}}{\lambda}\right] dt = 2\pi e^{-|y|/\lambda} . \qquad (5)$$

Equation (4) implies that if the probability for finding a vortex in an area $S_1 = L\lambda$ is low ($n\lambda L \ll 1$), the magnetic-field dependence of the average critical current is given by the standard Fraunhofer dependence $\overline{I_c} \sim j_0 L \, |(\pi\phi/\phi_0)^{-1}\sin(\pi\phi/\phi_0)|$ (Fig. 1a). Otherwise, calculating the integrals in Eq. (4) for $n\lambda L \gg 1$, we find

$$\overline{I_c^2} = 2j_0^2 L n\lambda\alpha_1 \{(n\lambda\alpha_1)^2 + [2\pi(\frac{\phi}{\phi_0 L} - \alpha_2 n\lambda)]^2\}^{-1} ;$$
$$1/\lambda L \ll n \ll 1/\lambda^2 . \qquad (6)$$

In this concentration range of the Abrikosov vortices the oscillation amplitude of the average critical current is exponentially small: $\sim\exp(-Ln\lambda\alpha_1)$ (Fig. 1b).

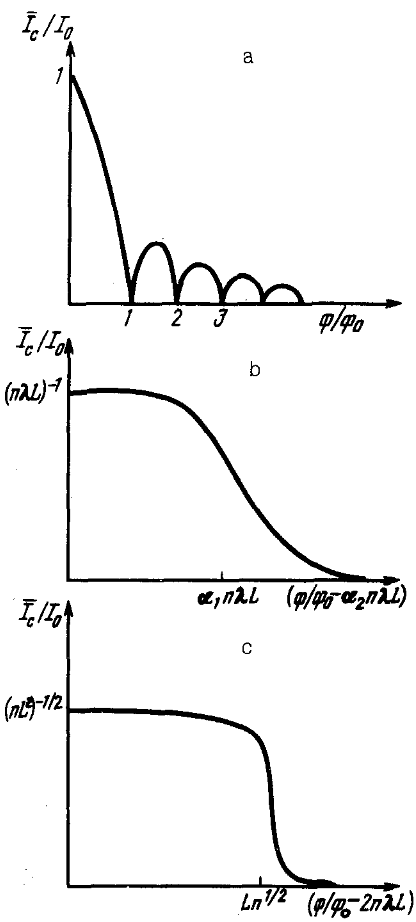


FIG. 1. Plot of the critical current vs the magnetic field for various concentrations of the Abrikosov vortices: (a) $n \ll 1/\lambda L$; (b) $1/\lambda L \ll n \ll 1/\lambda^2$; (c) $n \gg \lambda^{-2}$.

If the superconductors have many vortices, satisfying the condition $n\lambda^2 \gg 1$, the principal contribution in Eq. (3) to the current comes from approximately equal values of $x_1$ and $x_2$ ($|x_1 - x_2| \ll \lambda$), so that we can expand the argument of the exponential function in a series and consider only the first two terms. We then find

$$\overline{I_c^2} = j_0^2 \int_0^L dx_1 \times \int_0^L dx_2 \exp\{2\pi i(\frac{\phi}{\phi_0 L} - 2n\lambda)(x_1 - x_2) + 2\pi n(x_1 - x_2)^2 \ln|(x_1 - x_2)/\lambda|\}. \tag{7}$$

A departure in Eq. (7) from the Gaussian correlation function (the logarithm in the argument of the exponential function) stems from the presence of a singularity of the type $1/r$, where $r \to 0$, in the magnetic field of the Abrikosov vortices in the Josephson junction. Calculating the integrals over $x_1$ and $x_2$, we find the following expression in the region of the magnetic flux close to the value $2\phi_0 L n\lambda$:

$$\overline{I_c^2} = j_0^2 L \cdot \lambda\sqrt{\pi}(n\pi\lambda^2 \ln(n\pi\lambda^2))^{-1/2}\{1 - (\frac{\pi\phi}{\phi_0 L} - 2n\pi\lambda)^2 \frac{1}{2\pi n \ln n\pi\lambda^2}\},$$
$$|\frac{\phi}{\phi_0} - 2n\lambda L| \ll L(n \ln \pi n\lambda^2)^{1/2}. \tag{8}$$

If the magnetic flux $\phi$ differs drastically from $2\phi_0 L n\lambda$, the principal contribution to the integrals in Eq. (7) comes from $x_1$ and $x_2$, such that $(x_1 - x_2) \sim \phi_0 L/\phi$. As a result, the critical current begins to decrease rapidly with increasing magnetic field (Fig. 1c)

$$\overline{I_c^2} = \frac{j_0^2 L \pi^2 n}{(\frac{\phi}{\phi_0 L} - 2n\lambda)^3}. \tag{9}$$

This relation also holds for a small concentration of the Abrikosov vortices in magnetic fields $\phi\lambda/\phi_0 L \gg 1$.

Equations (6), (8), and (9) describe the dependence of the critical current on the magnetic field at various concentrations of the Abrikosov vortices. These equations differ considerably from the standard Fraunhofer dependence because the current density $j(x)$ correlates only at short range $x_0 \ll L$ ($x_0 \sim 1/n\lambda$ for $n\lambda^2 \ll 1$ and $x_0 \sim 1/\sqrt{n}$ for $n\lambda^2 \gg 1$), rather than on the entire plane of the junction. This circumstance accounts for the decrease in the critical current and for the change in its magnetic dependence.

The concentration of the pinned vortices and their positional correlation can thus be determined from the Josephson current plotted as a function of the magnetic field.

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