Coulomb blockade of tunneling in isolated junctions

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The Coulomb blockade of tunneling is analyzed systematically special emphasis is placed on the conditions for a blockade, which are required for further experiments.

The Coulomb blockade effect has been observed in junctions in which tunneling occurs through metal granules embedded in a dielectric layer.[1,2] The characteristic features seen on the current-voltage characteristics at bias voltages $V \sim e/C$ (C is the capacitance of a granule) and at temperature $T \lesssim eV$ are explained well by the theory of Ref. 3.

Recent technological advances have made it possible to fabricate tunnel junctions with an area $\sim 10^{-11}$ cm^2. There have been several reports[4–6] of the observation of a Coulomb blockade in junctions without granules, i.e., in isolated junctions. A semi-phenomenological theory (see the review[7]) has been derived on the basis of the concept of the capacitance of a junction, which is determined from the area of the junction and the thickness of the dielectric layer. The capacitance of the leads running to the junction is ignored; in any conceivable situation this capacitance would be many times that of the junction itself. A more accurate treatment of the Coulomb blockade in an isolated junction is required for future experiments, which have attracted interest partly because of their pertinence to technical applications.

The problem is to adequately incorporate the effect of quantum fluctuations of the electromagnetic field on the tunneling of a charge carrier. The key idea is that at $T=0$ the effect is determined by the electrical properties of the junction-lead system at frequencies $\sim eV/\hbar$.

We use the method proposed in Ref. 8 and not that in the case in which the material of which the leads are made has a metallic conductivity, with V on the order of the experimental values, we can ignore the retardation of the electromagnetic field, assuming that the electric potential is constant over distances on the order of the transverse dimension of a lead. We can also ignore the diffusion of charge out of the junction region. We then find the following result for the current-voltage characteristic at a constant voltage, in first order in the transparency of the junction:

$$I(V) = 4(\pi e R_T)^{-1} \operatorname{Im} \left\{ \int d\tau \tau^{-2} \sin^2(\omega\tau/2) \exp(-S(\tau)) \right\} \Big|_{\omega \to ieV+0}$$

$$S(\tau) = \int \frac{d\omega}{2\pi} \sin^2(\omega\tau/2) S(\omega); \qquad S(\omega) = e\varphi_\omega(0)/2|\omega| ,$$

The quantity φ_ω obeys a telegraphist's equation at imaginary frequencies:

$$\frac{\partial\varphi_\omega(x)}{\partial x} = R_0 i(x)\,; \qquad \frac{\partial i(x)}{\partial x} = C_0|\omega|\varphi_\omega(x)$$

$$i(0) = C|\omega|\varphi_\omega(0) - e.$$

Here C_0 and R_0 are the capacitance and resistance per unit length of the leads, C is the capacitance of the junction, and R_T is the resistance of the junction at $T \gg Ve$. At $T \neq 0$, the structural features on the current-voltage characteristic disappear for $eV \lesssim T$.

Assuming that the length of the leads, L, is infinite—a valid assumption under the condition $V \ll V_3 = \hbar/eC_0R_0L^2$—we find the following expression for $S(\omega)$:

$$S(\omega) = e^2/|\omega|(C|\omega| + (|\omega|C_0/R_0)^{1/2}) \ .$$

There is a substantial suppression of tunneling at values of V satisfying $eVS(eV)\hbar)/\hbar \gtrsim 1$. If the relation $V \gg \hbar C_0/R_0C^2e$, holds, the quantity $S(\omega)$ is dominated by the junction capacitance, and the predictions of the semiphenomenological theory hold. The I–V characteristic has the form $(V_1 = e/2C)$

$$R_T I = \begin{cases} V - V_1 \,\mathrm{sign}\, V & |V| > V_1 \\ 0 & |V| < V_1 \end{cases}$$

and the tunneling is blocked sharply at $V = V_1$.

In the opposite limiting case $V \ll \hbar C_0/R_0C^2e$ the quantity $S(\omega)$ is dominated by the leads, and the current-voltage characteristic is that predicted for the case of a long tunnel junction.[8] The suppression of the tunneling occurs in steps stretched out over an interval $\sim V_2 = e^3R_0/C_0\hbar$, in the limit $V \to 0$ we find $I \propto \exp(-V_2/4V)$. An analytic expression can be derived for the second derivative of the current with respect to the voltage:

$$\frac{\partial^2 I}{\partial V^2} R_0 V_2 = (4\pi(V/V_2)^3)^{-1/2} \exp(-V_2/4V) \ .$$

Two distinct types of Coulomb blockades thus exist.

When we allow the leads to be of finite length, we find a modification of $S(\omega)$ at low frequencies; the structural feature responsible for the suppression of tunneling is weakened. As a result, for a substantial suppression of tunneling the condition $V_3 \ll V_1$ or V_2 must hold.

We can summarize the discussion above in the following conditions:

1) If $R_1 = CR_0/C_0 \gg R_Q = \pi\hbar/e^2$, a blockade of the first type occurs.

2) If $R_1 \ll R_Q$, but if the total resistance of the leads satisfies $R_2 = R_0L \gg R_Q$, a blockade of the second type occurs at $V \lesssim V_2$.

3) If the conditions above do not hold, there will be no substantial suppression of tunneling, and the Coulomb effects will amount to no more than corrections.

How can condition 1 be met in practice? The leads are usually narrow metal films. If we wish to make the capacitance as small as possible, we could form a tunnel junction by putting such films together. The area of the junction would then be equal to the cross-sectional area of the lead. Condition 1 imposes a restriction on the conductivity of the material: $\sigma \ll e^2/a\hbar$, where $a \gtrsim 1$ Å is the effective thickness of the tunneling layer. This condition does not hold for good metals. If we overlap the leads, we lose ground—the capacitance of the junction is increased—but the limitation on the conductivity is relaxed: $\sigma \ll Ae^2a/\hbar$, where $A \gg 1$ is the ratio of the area of the overlap to the cross-sectional area of the lead.

The semiphenomenological theory predicts that at $V > V_1$, T the Coulomb blockade gives rise to a V-independent shift of the I–V characteristic along the voltage axis: $I = R_{(T)}^{-1} V \to I = R_{(T)}^{-1}(V - V_1 \text{sign } V)$. This shift is seen in all experiments and is interpreted as a precursor of a complete blockade of the tunneling at low V. The relations presented here show that this behavior of the current-voltage characteristic prevails at $V \gg \hbar C_0/R_0 C^2 e$, even if the conditions for the occurrence of a blockade do not hold. The shift of the current-voltage characteristic at high values of V and the suppression of tunneling at low values of V are thus phenomena which are not directly related to each other.

The reader might get the impression that the effects which we have been discussing here can be described only in electric-circuit terms. This is not the case at eV, $T \lesssim \hbar/\tau_{\text{diff}}$, where τ_{diff} is the time scale of the diffusion of the charge carriers over the length of the lead. This region has not yet been reached experimentally. At $T = 0$ and with $\hbar/\tau_{\text{diff}} \ll eV \ll eV_{1,2}$ we have $S(\omega) \sim (|\omega| R_2)^{-1}$, and the I–V characteristic behaves in accordance with $R_T I \sim V(V/V_{1,2})^{R_2/R_Q}$. At $\omega \sim \hbar/\tau_{\text{diff}}$, the expression for $S(\omega)$ becomes the final expression in the limit $\omega \to 0$, and at $eV \ll \hbar\tau_{\text{diff}}$, an Ohm's law with a greatly increased resistance is restored:

$$R_T I \sim V(\hbar/V_{1,2} e\tau_{\text{diff}})^{R_2/R_Q}.$$

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