Resonant-percolation paths as high-temperature superconducting channels in thin metal oxide films

V. Ya. Kirpichenkov
S. Ordzhonikidze Novocherkask Polytechnical Institute

(Submitted 22 December 1988)
Pis'ma Zh. Eksp. Teor. Fiz. **49**, No. 2, 116–119 (25 January 1989)

Ideas concerning one-dimensional resonant-percolation paths are used to demonstrate the possible occurrence of a size effect in this superconducting films. The effect takes qualitatively different forms in the cases of strong and weak coupling.

One of the major questions concerning high-temperature superconductivity remains unresolved: Which theoretical model—the traditional Bardeen–Cooper–Schrieffer (BCS) model or the various local-pair-superconductivity (LPS) models can successfully describe high-temperature superconductivity in the known metal oxides?[1] The reason is that experimentally it is extremely difficult to determine the value of the parameter t/U, which is a measure of the strength of the coupling [t is the width of the band, and U is the electron (or hole) attraction energy]. Under these circumstances it is worthwhile to seek conditions under which superconductors with different coupling strengths will behave in qualitatively different ways in order to resolve this question.

In the present letter we use ideas regarding resonant-percolation paths[2,3] to analyze one possibility for differentiating superconductors on the basis of the coupling strength. This possibility is based on a size effect which arises in systems that have dimensions which are finite along the direction in which the superconducting current flows: thin single-crystal films with a thickness $L \lesssim 1$ μm (or crystallites of ceramic high-temperature superconductors with a length scale L). For definiteness we will discuss $La_{2-x}Sr_xCuO_4$.

If, as is now generally believed, the stoichiometric composition La_2CuO_4 corresponds to a Mott insulator, then its doping with Sr most probably leads to the appearance in $La_{2-x}Sr_xCuO_4$ of O2p holes[4] (or hybrid Cu3d–O2p holes, but these distinctions are unimportant for the problem at hand). We will call "impurity centers" the unit cells into which Sr atoms penetrate and in which holes appear. We will assume that these impurity centers form a disordered system of identical centers, which are distributed at random among the cites of the three-dimensional lattice, in a manner which is, on the average, uniform over the volume with a relative concentration x (the model of structural disorder introduced by I. M. Lifshits), between which holes tunnel.

If the holes in a system which is infinite in three dimensions ($L \to \infty$) and in which the relation $x < x_c$ holds (x_c is the critical concentration of impurity centers, which corresponds to the percolation transition in a three-dimensional lattice) are localized either in local one-impurity levels with an energy E_0 (E_0 is the level ionization energy) and with a state localization radius $\sim \alpha^{-1}$ ($\alpha^2 = E_0$, $\hbar = 1$, $m = 1$) or in levels of finite clusters, then solitary resonant-percolation paths can form in systems with finite and sufficiently small values of L even at low concentrations $x \ll x_c$ in the disordered system of impurity centers. These solitary resonant-percolation paths are one-dimensional clusters of size $l(L) \gtrsim L$, which consist of chains of equidistant impurity centers which connect the opposite "banks" of the film. Energy bands of a resonant transparency form near the level E_0 along the resonant-percolation paths. The hole states in these bands "propagate" over the entire length of the resonant-percolation path. The conditions for the appearance of, and the statistical properties of, resonant-percolation paths of this sort were analyzed in detail in Refs. 2 and 3.

For the estimates below we assume that at all $x \leqslant x_c$, for the dimensions L in which we are interested here, the resonant-percolation paths are purely one-dimensional. In other words, we will ignore "hops" of holes between paths. This assumption is justified since even at $x = x_c$ the critical cluster in an infinite system is a one-dimensional formation. Furthermore, at the typical sizes of the paths, a fairly high probability for a Josephson tunneling between paths does not "build up".

Under these assumptions, as a rough criterion to tell us the temperature T_L at which a film of thickness L will go superconducting in the transverse direction it is natural to adopt the condition that the correlation radius of the superconducting short-range order of the one-dimensional system, $R_c(T_L)$, becomes on the order of the typical length of a resonant-percolation path, $l(L,x)$, which is on the order of the film thickness L:

$$R_c(T_L) \sim l(L, x) \sim L. \qquad (1)$$

Using the results of Refs. 5 and 6, we can write expressions for the correlation radius $R_c(T)$ in one-dimensional resonant-percolation paths in the limits of strong and weak coupling. We can put these expressions in a form convenient for further analysis.

In the case of strong coupling we write

$$R_c(T) = \frac{4\sin p_0}{\pi}\frac{t^2}{UT}d \sim \frac{t^2}{UT}d, \qquad t/U \ll 1, \qquad (2)$$

where d is the distance between impurity centers along a resonant-percolation path, p_0 is the Fermi momentum, t is the width of the energy band along the path, and U is the hole attraction energy at an impurity center. With a fixed average number of holes per impurity center the quantity $\sin p_0 = \text{const} \sim 1$ is the same for all paths, and it does not change when the other parameters of the paths are varied.

For the case of weak coupling we write

$$R_c(T) = \frac{\hbar^2 |\Psi_0|^2}{mT} \sim \frac{\Delta^2(T)\, t}{T_c^2\, T}d, \qquad t/U \gg 1, \qquad (3)$$

where $\Psi_0 \sim n^{1/2}\, \Delta(T)/T_c$ is the equilibrium value of the order parameter, n is the linear concentration of holes along the path, and

$$\Delta(T) \sim \begin{cases} T_c & T \ll T_c \\ [T_c(T_c - T)]^{1/2}, & T \lesssim T_c, \end{cases} \tag{4}$$

$$T_c \sim t\exp(-t/U). \tag{5}$$

Using condition (1) and Eqs. (2)–(5), we find,

$$T_L^{\rm LPS} \sim \frac{d}{L}\frac{t^2}{U} \tag{6}$$

$$T_L^{\rm BCS} \sim \begin{cases} \dfrac{d}{L}T & , \quad L \gg \dfrac{t}{T_c}d & \text{(7a)} \\ (1 - \dfrac{T_c L}{td})T_c \approx T_c, & L \ll \dfrac{t}{T_c}d & \text{(7b)} \end{cases}$$

Since the parameters d and t are random at fixed values of the film thickness L and the concentration x, Eqs. (6), (7a), and (7b) must be averaged over the set of paths. We turn now to a qualitative discussion of the corollaries of these equations at various values of x.

1. At $x = x_c$, the energy width of the typical paths, $\langle t \rangle$, does not depend (or depends only weakly) on L. In this case we see from (6) and (7b) a qualitative difference in the behavior of $\langle T_L \rangle$ as a function of L at small L. Specifically, while we have $\langle T_L^{\rm LPS} \rangle \sim {\rm LO}^{-1}$ over the entire length interval, we have $\langle T_L^{\rm BCS} \rangle \to \langle T_c \rangle$, and this quantity is nearly independent of L in the asymptotic region in (7b).

2. Under the condition $(x_c - x)/x_c \ll 1$ we have, in accordance with percolation scaling concepts[3] and at a logarithmic accuracy,

$$-(\alpha L)^{-1}\ln(\langle t \rangle / E_{0'}) \sim (x_c - x)^{\nu}. \tag{8}$$

We then see from (5) that in length region (7b) the quantity $\langle T_L^{\rm BCS} \rangle$ in fact decreases with decreasing L. We note in this connection that in their review Gabovich and Moiseev[7] discuss some experiments in which the superconducting transition temperature was observed to decrease upon a decrease in the film thickness, $L \lesssim 4000$ Å, and during a corresponding size effect upon the crushing of a ceramic.

3. At $x \ll x_c$ the energy widths and probabilities for the occurrence of resonant-percolation paths are extremely small, so it is difficult to observe superconductivity experimentally. At such concentrations the parameter $\langle t \rangle / U$ can cross unity, so the type of superconductivity can change.

4. In the region $x > x_c$, there may be either a one-dimensional or quasi-one-dimensional situation, depending on the relation between the typical size of the "voids" in the infinite cluster of impurity centers and the film thickness.

Numerical estimates show that this size effect should be observed in a small neighborhood of x_c in the length interval $L \sim 10^4$–10^2 Å.

[1]L. P. Gor'kov and N. B. Kopnin, Usp. Fiz. Nauk **156**, 117 (1988) [Sov. Phys. Usp. (in press)].
[2]I. M. Lifshitz and V. Ya. Kirpichenkov, Zh. Eksp. Teor. Fiz. **77**, 989 (1979) [Sov. Phys. JETP **50**, 499 (1979)].
[3]I. M. Lifshitz, S. A. Gredeskul, and L. A. Pastur, Zh. Eksp. Teor. Fiz. **83**, 2362 (1982) [Sov. Phys. JETP **56**, 1370 (1982)].
[4]G. M. Éliashberg Pis'ma Zh. Eksp. Teor. Fiz. **48**, 275 (1988) [JETP Lett. **48**, 305 (1988)].
[5]T. M. Rice Phys. Rev. **A140**, 1889 (1965).
[6]K. B. Efetov and A. I. Larkin, Zh. Eksp. Teor. Fiz. **69**, 764 (1975) [Sov. Phys. JETP **42**, 390 (1975)].
[7]A. M. Gabovich and D. P. Moiseev, Usp. Fiz. Nauk **150**, 599 (1986) [Sov. Phys. Usp. **29**, 1135 (1986)].

Translated by Dave Parsons