Asymptotic finiteness and supersymmetry in quantum field theory

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Grand unified theories which contain scalars, spinors, and gauge fields whose multiplet composition allows a finiteness in terms of all coupling constants are analyzed. In the UV limit, such theories are asymptotically finite, while in the IR limit they are asymptotically supersymmetric and finite.

1. Some progress has recently been made in constructing finite field theories, i.e., theories in which the β functions of the coupling constants and the masses are zero. In particular, realistic supersymmetric finite grand unified theories[1–4] based on, for example, the SU(5) group,[1–3] have been constructed.

In order to make a theory finite, one must impose some rigid restrictions on both the multiplet composition of the theory and the values of the coupling constants. If the conditions on the coupling constants are violated, the theory ceases to be finite, but it may retain a multiplicative renormalizability. In this case a renormalization-group method can be used to study the behavior of the effective charges corresponding to the coupling constants. As we show below, the theory then becomes finite in the UV limit, while in the IR limit it becomes supersymmetric and finite.

2. We use the single-loop approximation to analyze the asymptotic behavior of the effective coupling constants in massless grand unified theories which contain a single gauge coupling constant g, several Yukawa constants h, and several scalar coupling constants f (we will be omitting the indices everywhere). Under certain restrictions on the multiplet composition of the scalars and the spinors (Ref. 4, for example), the theory is finite in terms of the constant q. In this case we have $g(t) = g$, where t is the renormalization-group parameter; the limit $t \to \infty$ corresponds to the UV asymptotic region, and $t \to -\infty$ corresponds to the IR asymptotic limit.

The equation for the effective Yukawa coupling constants has the structure[5,6]

$$dh/dt = a_1 h(h^2 - a_2 g^2), \tag{1}$$

where a_1 and a_2 are constants. As a rule, there are two fixed points for h; $\bar{h}_1 = 0$, $\bar{h}_2 \neq 0$. In the limit $t \to \infty$, the zeroth fixed point is stable,[7] while in the limit $t = -\infty$ the nonzeroth fixed point is stable. If the grand unified theory under consideration is supersymmetric at $h \sim g$, $f \sim g^2$, the point $\bar{h}_2$ may correspond to a supersymmetric theory (cf. Ref. 4).

The finiteness conditions for scalar coupling constants (the equations for the corresponding fixed points) are

$$k_1 f^2 + k_2 f g^2 + k_3 g^4 + k_4 f h^2 + k_5 h^2 = 0, \quad k_i = \text{const}, \quad i = 1, \dots, 5. \tag{2}$$

For the groups which are of interest from the phenomenological standpoint [e.g., SU(N) with $N\leqslant 5$], Eqs. (2) generally have real solutions only if $h = h_2 \neq 0$ (Refs. 4–6).

In the limit $t \to \infty$ there are two possibilities.[8] If $h(0)$ has an arbitrary value in the theory, we find $h(t) \to 0$ in the limit $t \to \infty$ (Ref. 7). Equations (2) then have no real solutions, and we find $|f(t)| \to \infty$. We can fix the initial values by requiring $h(0) = \bar{h}_2$. In this case we have $h(t) \equiv \bar{h}_2$, and Eqs. (2) have real solutions.[8]

If the multiplet structure of the theory allows a supersymmetry, one of the solutions of Eqs. (2) generally corresponds to a supersymmetric theory. We denote this solution by f_1, and we denote the other solutions (if they exist) by f_2. For several models [e.g., an SU(2) gauge theory with a global SU(4) or SU(2)$\times$SU(4)$\times$U(1) symmetry and an SU(5) grand unified theory], which were cited in Refs. 4 and 3, it can be shown that in the limit $t \to \infty$ the nonsupersymmetric solutions f_2 are stable, while in the limit $t \to -\infty$ the supersymmetric solutions f_1 are stable. It can be assumed that for all models of this type f_2 is stable in the limit $t \to \infty$, and f_1 in the limit $t \to -\infty$.

In the limit $t \to \infty$ the theory thus has an asymptotic finiteness by virtue of the special choice of initial condition on the Yukawa constant, $h(0) = \bar{h}_2$. In other words, the effective charges $h(t)$ and $f(t)$ tend toward the values which correspond to a finite nonsupersymmetric theory. The initial values of the scalar constants are arbitrary.

In the IR limit ($t \to -\infty$) we find $h(t) \to \bar{h}_2$ for arbitrary initial values $h(0)$ and $f(0)$, and we also find $f(t) \to f_1$. In the IR limit this theory thus becomes asymptotically finite and asymptotically supersymmetric. The asymptotic supersymmetry is apparently realized in the IR limit (and not in the UV limit) because the violation of supersymmetry is rigid.

3. We now consider a grand unified theory in which the finiteness conditions are resolvable in all orders of perturbation theory with $h = \bar{h}_2, f = f_1$. In other words, the multiplet structure of the theory is of such a nature that with $h = \bar{h}_2, f = f_1$ the exact β functions of all of the effective coupling constants vanish. One such theory is an SU(2) gauge theory with a global SU(4) invariance,[4] in which there are two scalar coupling constants and one Yukawa constant. If all of these coupling constants (in the notation of Ref. 4) are equal to the gauge constant g, we find an $N = 4$ expanded supersymmetric gauge theory which is finite in all orders of perturbation theory.

The exact β functions for theories of this type have a fixed point at $h = \bar{h}_2, f = f_1$. Furthermore, all (or some) of the effective charges may have infinite asymptotic expressions. The single-loop analysis (§2) suggests that the fixed point $h = \bar{h}_2, f = f_1$ is stable in the IR limit. In other words, this model is asymptotically finite and asymptotically supersymmetric in the limit $t \to -\infty$ outside perturbation theory.

In the UV limit the effective charges will apparently tend toward infinity. The results of §2, however, suggest the following hypothesis: Under certain conditions, a nonsupersymmetric fixed point $h = \bar{h}_2, f = f_2$ for the exact β functions, which is stable in the limit $t \to \infty$, may exist. This circumstance means that there are nonsupersymmetric solutions of the finiteness conditions in all orders of perturbation theory.

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