Ion-charge scaling law for electron and optical spectra

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The effect of strong coupling on the optical properties, the ionization composition, the relaxation of a deviation from equilibrium, and the negative-energy electron spectrum is the same in a nondegenerate plasma of singly charged ions with an electron density N_e and a temperature T as in a plasma of ions of charge z with a density z^4N_e and a temperature z^2T.

In a large number of plasma-physics systems, one can see effects of a strong coupling, which arise because the plasma acquires several features which are characteristic of condensed media and which lead to multiple changes in the plasma properties. For the ionization composition, for the relaxation of a deviation from equilibrium, and for the optical properties, these effects are explained in terms of a perturbation of the negative electron energy spectrum, $E_e < 0$. Significant progress has been achieved (Refs. 1 and 2, for example) in the theory of these effects for the $z = 1$ case. There has been less study of dense plasmas with $z > 1$, although such plasmas are encountered in applications (plasma lasers; intense explosions; fast electron, neutron, and optical heating; high-velocity impact; intense discharges; etc.). The complexity of carrying out measurements in such systems means that the plasma properties must be calculated from models which stand on shaky ground (e.g., the model of an ideal plasma).[3–5]

In the present study we have observed a qualitative similarity in these strong-coupling effects in plasmas with $z = 1$ and $z > 1$, and we have established a law for determining the influence of these effects at a quantitative level.

1. The state of an electron with $E_e < 0$ in a plasma—a low-density, disordered "lattice" of slow ions—is determined, as we know, by the ratios among some quantities which are of the nature of energies: the overlap integral J, the amplitude (U) of the long-wavelength potential fluctuations, the spectral distance (Δ) between neighboring levels which can be involved in the formation of a cluster, the Stark broadening δ, the Inglis-Teller optical shift of the photoionization thresholds (E_{IT}) etc. In addition to the bound states which are localized in the Lifshitz sense (i.e., single-center states, with $4J < \Delta$) and in the Anderson sense (i.e., in a potential well, with $J < U$), which constitute zone 1 in (Fig. 1), the following may form: zone 2, which consists of levels which are broadened to the point that they coalesce into a quasicontinuum ($\delta > \Delta$); zones 3 and 4, which correspond to electrons which are localized in a potential well but which are delocalized to the extent that the ions cluster ($4J > \Delta$ and $J < U$; zone 4 is associated with a macroscopic localization region); zone 5, which consists of quasifree electrons, which are moving in regions with a lowered potential that permeates the entire plasma volume ($|E_e| < E_f$); and zone 6, which consists of states which are similar to conduction electrons of a liquid metal ($4J > \Delta$ and $J > U$).

The effect of strong coupling on the plasma properties which we listed above is

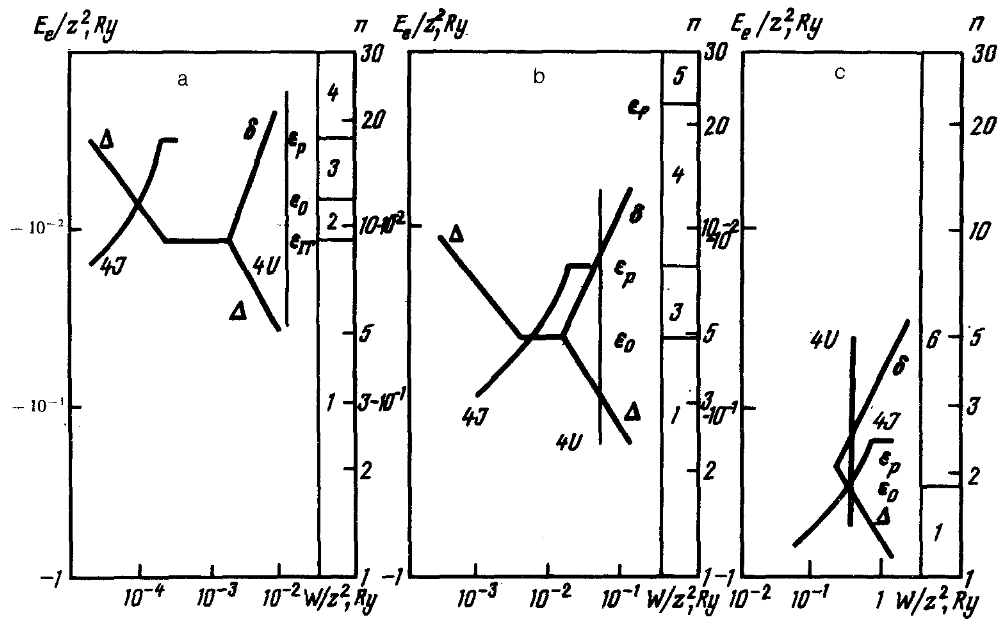

FIG. 1. Electron spectra. a—Slight strong-coupling effects ($N_e = 0.5\times10^{16} z^4$ cm^{-3}); b—strong coupling ($N_e = 0.5\times10^{18} z^4$ cm^{-3}); c—liquid-metal plasma ($N_e = 0.5\times10^{21} z^4$ cm^{-3}), $T = z^2$ eV.

associated with these zones. The presence of zone 2 (Fig. 1a), for example, makes the coefficient (κ) of the continuum absorption of light higher than the ideal value κ_i by an average factor of $\exp(E_{IT}/T)$, while in the absence of this zone (Fig. 1b) the ratio $\zeta = \kappa/\kappa_i$ is reduced by a large factor because of the appearance of a "soft gap," as in the spectrum of a solid, i.e., a decrease by a factor of g/g_0 in the population of zones 3 and 4 (g is the effective statistical weight, and g_0 the unperturbed statistical weight). For a plasma of z-ions we can derive the following in the hydrogen-like approximation:

$$\begin{aligned}
&J(\mathbf{r}_j - \mathbf{r}_{j'}) = \int \psi^*_{n'l'm'}(\mathbf{r} - \mathbf{r}_{j'})(ze^2/|\mathbf{r}_j - \mathbf{r}|)\,\psi_{nlm}(\mathbf{r} - \mathbf{r}_j)\,d^3\mathbf{r};\\
&\Delta = 2\mathrm{Ry}\,z^2/n^3 \ \text{ for } \ |E_e| > E_{IT},\ \Delta = 2\mathrm{Ry}\,z^2/n^4 \ \text{ for } \ |E_e| < E_{IT};\\
&\delta = 3n^2 e a_0 F_H/z,\ F_H = (4\pi/3)^{2/3} ez N_i^{2/3}; \qquad (1)\\
&E_p = 2^{2/3} z e^2 N_i^{2/3};\ U = e^{3/2}(N_i T(z + z^2))^{1/4};\ E_f = 0{,}2U;\\
&E_{IT} = z^2\,\mathrm{Ry}\,(3e^2 a_0 (4\pi/3)^{2/3} N_i^{2/3}/(2\mathrm{Ry}\,z^2))^{2/5}.
\end{aligned}$$

Here $\psi_{nlm}(x) = z^{2/2} F_{nlm}(zx)$is the wave function of the state: F_{nlm} is a function whose form depends on the quantum numbers n, l, and m; $\mathbf{r}_j$, $\mathbf{r}_{j'}$, and $\mathbf{r}$ are the radius vectors of the nuclei of neighboring ions and the given radius vector; Ry = 13.6 eV; α_0 is the first Bohr radius; and ψ^* is the complex conjugate of ψ.

These parameters depend on z in different ways, and as z increases we can expect

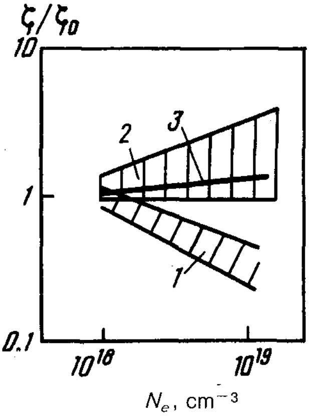

FIG. 2. Effect of strong coupling on the optical absorption continuum; here ζ is divided by $\zeta_0 = \zeta(N_{e0} = 10^{18}\ \mathrm{cm}^{-3})$. 1: Data of Refs. 1 and 2 for $z = 1$. 2: Experimental results on C_2F_4 and for the Cu plasma of the focus of a plasmadynamic discharge of a magnetoplasma compressor[6]; $z = 2$, $T = 4.5$–5 eV. The ζ/ζ_0 regions found from measurements by various methods are shown: the absorption coefficients at wavelengths of 362.8 nm and 441.6 nm; the absorption spectra at photon energies of 2–6 eV; the emission spectra at 0.5–6 eV and 10–350 eV; and the emission fluxes integrated over the spectrum over the intervals 12–65, 6–250, and 11.5–250 eV. 3: Calculation from expressions (1) and (2).

some new combinations of inequalities, i.e., the appearance of some qualitatively new electron states. It would seem that experiments also point in this direction. With $N_e = 10^{18}$–$10^{19}\ \mathrm{cm}^{-3}$ for $z > 1$, for example, instead of the decrease in ζ which would be characteristic of a singly charged plasma (because of the soft gap) one observes an increase in ζ (Fig. 2).[6]

2. However, we will take the approach of scaling theory. The quantities J, Δ, δ, and E_{IT} are integrals of solutions of the Schrödinger equation for an electron in a potential well, which is determined by the fields of the various systems of Coulomb centers ze, against a quasicontinuous background of a nondegenerate electron gas. It can be shown that there is a scaling in z for this equation if the linear dimensions transform in inverse proportion to z, while the energy quantities W transform in proportion to z^2, i.e.,

$$W(z, E_e, T, N_i) = z^2 W(1, E_e/z^2, T/z^2, N_i/z^3). \tag{2}$$

A simple substitution[1)] shows that (2) is valid for $W = J$, Δ, δ, E_{IT}, and E_p. For U and E_f relation (2) holds within $\approx z^{1/4}$; relation (2) is also valid for non-hydrogenlike ions. For example, estimates incorporating the quadratic Stark effect (on the basis of the equations of Ref. 3) describe a systematic deviation of only $\approx z^{1/3}$.

3. It follows that the energy diagram (Fig. 1) for a singly charged plasma with N_{e1} is essentially identical to that for a z-charge plasma with $N_e = z^4 N_{e1}$; i.e., qualitatively new states do not appear, so we could expect the formation of a soft gap and related effects. Any state of the plasma with the parameters z', and $N'_{e'}$ and T' corresponds to several similar states (similar in terms of the effect of strong coupling on these properties) of a z''-charge plasma with $N''_e = (z''/z')^4 N'_e$ and $T'' = (z''/z')^2 T'$, for which the values of g/g_0, E_{IT}/T, etc., are identical. The properties of a plasma with $z > 1$ can therefore be calculated from the data for $z = 1$ As z increases, the strong coupling becomes manifested at higher values of N_e, because of the small dimensions of multiply charged ions.

These conclusions agree with experiments. For example, the results calculated from (1) and (2) correspond to the data from Ref. 6 (Fig. 2). For the states with moderate values of N_e and $z > 1$, which have been studied in most detail, calculations based on Refs. 3–5 and 7 are consistent with experimental data. According to (2), a soft gap has not yet formed there, and the effects associated with a soft gap should substantially change the properties of the plasma at slightly higher values of N_e, which pose greater difficulties for diagnostics but which are often important for applications.[3,7,8]

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1) For example, $E_{IT}(z,N_i) = z^2\mathrm{Ry}(3e^2a_0(4\pi/3)^{2/3}(N_i/z^3)^{2/3}/(2\mathrm{Ry}))^{2/5} = z^2E_{IT}(1,N_i/z^3)$.

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