Longitudinal magnetothermal emf of metal with an arbitrary Fermi surface

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The longitudinal component of the thermal emf tensor of a metal in a magnetic field is found to experience giant oscillations.

It is well known that the kinetic characteristics of a metal, depending on the strength of the magnetic field H applied to it, experience oscillations of the same type as those occurring in the de Haas-van Alphen effect. If, however, the de Haas-van Alphen oscillations are related to the periodic dependence of the thermodynamic potential Ω (μ) on the field H, oscillations of the kinetic characteristics will occur because of the changes in the scattering of the conduction electrons in the magnetic field. It was shown in Refs. 2 and 3 that the oscillating increment in the conductivity occurs as a result of the change in the probability for the scattering of electrons by impurities in the magnetic field. The magnetothermal emf has been studied until now, however, either on the basis of a thermodynamic method[4] or as a first approximation with respect to scattering,[5] while the dependence of the electron-impurity relaxation time τ on the magnetic field has been ignored.

In the present letter we analyze the oscillations of longitudinal magnetothermal emf of a metal with an arbitrary Fermi surface, which result from the dependence of the conduction-electron relaxation time on the field H. We will restrict the analysis to the semiclassical case $\Omega \ll \mu$, where μ is the chemical potential, and Ω is the cyclotron frequency, but we will assume that $\tau \gg \Omega^{-1}$. If the magnetic field H is directed along the z axis, the electron energy is given by the equation for the Landau levels

$$\epsilon_\sigma (p_z, n) = \frac{S(p_z, n)}{2\pi m^*} - \frac{eH}{2mc}\hat{\sigma}_z + E(p_z), \tag{1}$$

where $S(n,p_z) = (2\pi eH/c)(n+\gamma)$ is the cross-sectional area of the constant-energy surface in the plane $p_z = \text{const}$, $m^* = (2\pi)^{-1}(\partial s/\partial \epsilon)$ is the cyclotron mass, $\Omega = eH/m^*c$, $\hat{\sigma}_z$ is the Pauli matrix, and $\gamma \in [0,1]$.

Let us consider the case in which the metal is pure ($T\tau \gg 1$), assuming, for simplicity, that the matrix element of the scattering, U, is a constant. In the semiclassical limit, where the states are characterized by the quantum numbers n and p_z, we find the following expression for the scattering probability:

$$W(\epsilon) = \frac{n_i |U|^2 eH}{2\pi c} \int dp'_z \sum_{n'} \delta(\epsilon - \epsilon_\sigma(p'_z, n')). \tag{2}$$

If the interaction of electrons with the impurity is assumed to be independent of the electron spin, the spin projection does not change during the scattering and the corresponding currents of electrons with spin projections $\pm 1/2$ can be considered separately. Since the kinetics of the metal is determined exclusively by the electrons near the Fermi surface ($\xi = \epsilon_\sigma - \mu \lesssim T$), and since the factor $\cosh^{-2}[(\epsilon_\sigma - \mu)/2T]$ always appears in the integral expressions for the kinetic coefficients, we can formally include the spin dependence of ϵ in the chemical potential

$$\epsilon_\sigma(p_z, n) - \mu = \epsilon(p_z, n) - \mu_\sigma , \quad \mu_\sigma = \mu + \frac{eH}{2mc}\hat{\sigma}_z . \tag{3}$$

The expression for the scattering probability $W(\epsilon)$, transformed by means of the Poisson formula, becomes

$$W(\epsilon) = W_0[1 + \frac{eH}{\pi^2 c\nu(\epsilon)} \mathrm{Re} \int dp_z' \sum_{k=1}^{\infty} \int_{-\gamma}^{\infty} \delta(\epsilon - \epsilon(p_z', n')) e^{2\pi i k n'} dn'] , \tag{4}$$

where $W_0 = \pi n_i |U|^2 \nu(\epsilon)$ is the scattering probability which does not depend on the field H.

Changing in a standard fashion from integration over n' to integration over the energy, expanding $n(\epsilon, p_z)$ in powers of p_z near the points of the extreme value of $n_m(\epsilon)$, and, finally, integrating over p_z' and ϵ' for the relaxation time τ, we find

$$\tau(\epsilon) = \tau_0 [1 - \frac{1}{\pi^2 \nu(\epsilon)} (\frac{2\pi eH}{c})^{1/2} \mathrm{Re} \sum_m m_m^* e^{i\eta\frac{\pi}{4}} \left| \frac{\partial^2 S_m}{\partial p_z^2} \right|^{-1/2}$$

$$\times \sum_{k=1}^{\infty} k^{-1/2} e^{2\pi i k [n_m(\mu_\sigma) + (\partial n/\partial\epsilon)_{\mu_\sigma}(\epsilon - \mu_\sigma)]}], \tag{5}$$

where $\tau_0 = W_0^{-1}$. Summation over m means summation over all of the extreme cross sections of the Fermi surface in the plane $p_z = \mathrm{const}$, and $\eta = \mathrm{sign}(\partial^2 S_m/\partial p_z^2)$.

Let us calculate the component of the β_{zz} tensor of the thermoelectric coefficient β_{ik} ($\nabla T \| \mathbf{E} \| \mathbf{H} \|$ to the z axis)

$$\beta_{zz} = - \frac{e}{3T} \int v^2 \tau(\xi) \nu(\xi) \frac{\xi d\xi}{4T \cosh^2 \xi/2T} . \tag{6}$$

Substituting expression (5) in (6), integrating over ξ, and averging over the spin projections, we find

$$\beta_{zz} = \beta_{zz}^0 + \beta_{zz}^H , \tag{7}$$

where $\beta_{zz}^0 = -(e\pi^2/9)T\{[d(v^2\tau_0\nu)/d\mu]\}\mu$ is the thermoelectric coefficient β in the absence of a magnetic field, and

$$\beta_{zz}^{H} = -\frac{1}{3}\left(\frac{2}{\pi}\right)^{3/2} e(v^2\tau_0)_{\mu}\left(\frac{eH}{c}\right)^{1/2} \sum_m m_m^* \left|\frac{\partial^2 S_m}{\partial p_z^2}\right|^{-1/2} \sum_k k^{-1/2}\cos\left(\pi k\frac{m_m^*}{m}\right)$$

$$\times\ \Psi_1\left(\frac{2\pi^2 T}{\Omega}k\right)\sin\left[\frac{S_m c}{eH}k + \eta\frac{\pi}{4}\right]. \qquad (8)$$

In contrast with the background part of β_0, in the calculation of integral (6) the principal frequency dependence (in the parameter μ/Ω) was found to be attributable to the rapidly oscillating exponential function, rather than to the standard term $\sim\xi^2$. Interestingly, the function included in the result

$$\Psi_1(x) = \int_0^\infty \frac{t\sin\frac{2x}{\pi}t}{\cosh^2 t}dt = \frac{\pi}{2\sinh x}[x\coth x - 1] = \begin{cases}\frac{\pi}{6}x\left(1-\frac{7}{30}x^2\right) & x \ll 1,\\ \pi x e^{-x} & x \gg 1\end{cases}, \qquad (9)$$

is the total derivative of the function $\Psi(x) = x/\sinh x (\Psi_1(x) = -(\pi/2)\Psi'(x))$ which arises in the description of the oscillations of the Shubnikov-de Haas conductivity of the normal metal in a magnetic field.[1]

The behavior of the longitudinal magnetothermal emf, $Q_{zz}(\Omega) = -\beta_{zz}/\sigma_{zz}$ coincides with the behavior of the thermoelectric coefficient β_{zz} within terms of order $\sim\Omega/\mu$. The relative oscillations of the conductivity $(\sigma_{zz}-\sigma_{zz}^0)/\sigma_{zz}^0$ in a magnetic field have the form

$$\frac{\sigma_{zz}-\sigma_{zz}^0}{\sigma_{zz}^0} = \frac{1}{\pi^{3/2}\nu(\mu)}\left(\frac{2eH}{c}\right)^{1/2}\sum_m m_m^*\left|\frac{\partial^2 S_m}{\partial p_z^2}\right|^{-1}\sum_k k^{-1/2}\cos\left(\pi k\frac{m_m^*}{m}\right)\Psi\left(\frac{2\pi^2 T}{\Omega}k\right)$$

$$\times\ \cos\left[\frac{S_m c}{eH}k + \eta\frac{\pi}{4}\right]. \qquad (10)$$

It is easy to see that these oscillations in the parameter Ω/μ are small compared with the oscillations $\beta_{zz}^H/\beta_{zz}^0$, with which they do not coincide in phase, since $\Omega_{zz}(\Omega) \approx -\beta_{zz}^H/\sigma_{zz}^0$.

For clarity, further analysis will be carried out on the basis of a model for free electrons $[\epsilon = \Omega(n+1/2) + p_z^2/2m, m^* = m, (\partial^2 s/\partial p_z^2) = -2\pi]$, where expression (8) is simplified considerably

$$\beta_{zz}^H = (6/\pi^2)\beta_{zz}^0(\mu/2T)^{1/2}(\Omega/T)^{1/2}$$

$$\times\sum_k(-1)^{k+1}k^{-1/2}\Psi_1\left(\frac{2\pi^2 T}{\Omega}k\right)\sin\left(\frac{2\pi\mu}{\Omega}k - \frac{\pi}{4}\right). \qquad (11)$$

In the case of weak fields ($\Omega \ll 2\pi^2 T$), because the terms of the series decrease exponentially, we can use in (11) only the first term of the sum with $k = 1$, and for the thermoelectric coefficient we find

$$\beta^H_{zz} = 6\pi\beta_0 (2\mu/\Omega)^{1/2} \exp\left(-\frac{2\pi^2 T}{\Omega}\right) \sin\left(\frac{2\pi\mu}{\Omega} - \frac{\pi}{4}\right). \qquad (12)$$

In the case of strong fields ($\Omega \gg 2\pi^2 T$) we can estimate the sum in (11) by using only the terms with $k \lesssim k_0 \sim (\Omega/2\pi^2 T)$ in the summation of the series and by replacing the function $\Psi_1(2\pi^2 T/\Omega)$ with its asymptotic value in the case of small arguments.

$$s = \sum_{k=1}^{\infty} (-1)^k k^{-1/2} \Psi_1\left(\frac{2\pi^2 T}{\Omega} k\right) \sin\left(\frac{2\pi\mu}{\Omega} k - \frac{\pi}{4}\right) \approx \frac{\sqrt{2}\pi^3}{6} \frac{T}{\Omega} \sum_{k=1}^{\sim k_0} (-1)^k k^{1/2}$$
$$\times \left[\sin\frac{2\pi\mu}{\Omega} k - \cos\frac{2\pi\mu}{\Omega} k\right]. \qquad (13)$$

At the points $\Omega_{N+1/2}$ such that $\mu/\Omega_{N+1/2} = N + 1/2$ (N is an integer), where all the terms of the sum in (13) have the same sign $\beta^H(\Omega_{N+1/2}) \sim \beta_0(\mu/T)^{1/2}(\Omega_{N+1/2}/T)$. The value of the thermoelectric coefficient can also be determined at the other singular point $\Omega = \Omega_{N+1/4}$. Here the expression enclosed in square brackets in (13) has the

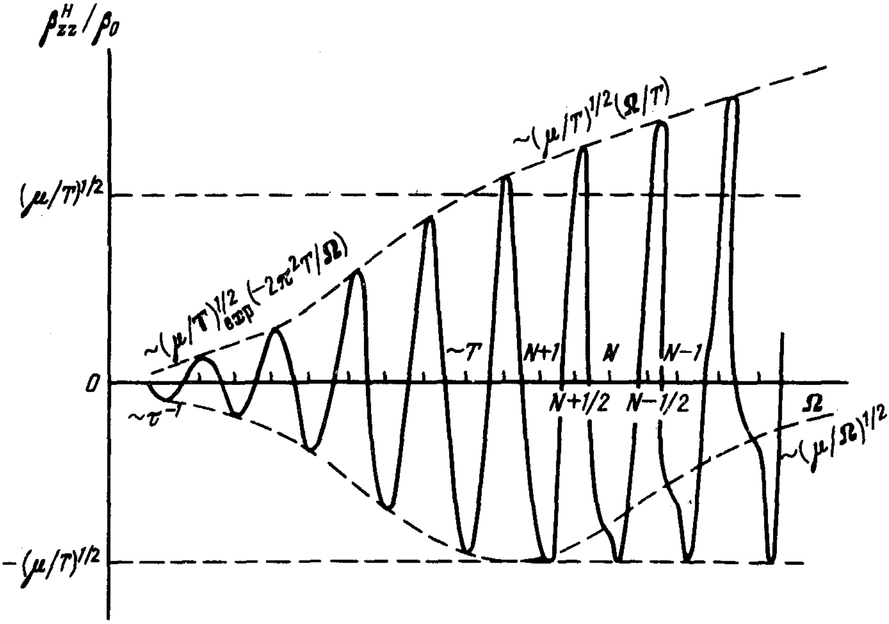

FIG. 1. Relative thermoelectric coefficient β^H_{zz}/β^0 plotted schematically as a function of the cyclotron frequency $\Omega = eH/m^*c$. The points $N + a$ represent the values of Ω_{N+a}, for which $\mu/\Omega_{N+a} = N + a$, where N is an integer.

value of ± 1 and the sum can be calculated. This sum is defined by its first terms [rather than by the upper limit, as was the case for $s(\Omega_{N+1/2})$]; $\beta^H(\Omega_{N+1/4}) = -0.65\beta_0(\mu/\Omega_{N+1/4})^{1/2}$. A similar analysis of the sum in (13) at the points Ω_N and $\Omega_{N-1/4}$ shows that $\beta(\Omega_N) \sim \beta(\Omega_{N-1/4}) \sim \beta_0(\mu/T)^{1/2}$, but the sign of this sum changes rapidly changing Ω.

We note that β^H is larger than β_0 with respect to the large parameter μ/T. In this respect, we can say that giant oscillations arise in the longitudinal thermal emf in a strong magnetic field $(\Omega \gg 2\pi^2 T)$.

The oscillations of the coefficient β_{zz}^H which set in with increasing magnetic field are shown schematically in Fig. 1. Sinusoidal oscillations with a gradually increasing amplitude occur in weak fields. In strong fields, however, the curve becomes strongly asymmetric with respect to the Ω axis, displays peaks in the region $\beta > 0$, which correspond to the points $\mu/N + 1/2$, and at the points $\mu/N + 1/4$, $\beta < 0$ it has an inflection which increases markedly with increasing field.

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