Quantum fluctuations in disordered magnetic materials

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A spatial disorder in magnetic materials in which the order parameter is not conserved leads to spin cancellations even in the approximation of a self-consistent field. These corrections may be large in antiferromagnets with a spin $S = 1/2$ if the disorder is sufficiently pronounced.

In systems with a nonconserved order parameter the decrease in the order parameter due to quantum fluctuations becomes smaller as the interaction radius r_0, increases, and it vanishes and advantages in the approximation of a self-consistent field. We will show here that when a disorder is introduced in a magnetically ordered system, some new quantum effects arise. Frustrations lead to spin cancellations even in the approximation of a self-consistent field; i.e., the quantum corrections to the order parameter do not contain an ordinary small parameter of the type $1/z$, where z is the number of neighbors. In highly disordered systems, the spin cancellations may thus be significant.

We consider a two-sublattice Heisenberg antiferromagnet in which the exchange interaction between sublattices, $\tilde{V}_{ij}$, fluctuates:

$$\tilde{V}_{ij} = V_{ij} + J_{ij}, \qquad \bar{J}_{ij} = 0, \qquad \overline{J_{ij}^2} = D_{ij}^{ex} . \tag{1}$$

The superior bar means a configurational average. At absolute zero, the disorder correction to the energy of the system is then

$$\delta F = -\frac{1}{16\pi}\int d\omega \sum_{i,j} D_{ij}^{ex}(K_{i_1,j_2}(\omega) + K_{j_2,i_1}(\omega) + K_{j_2,j_2}(\omega) + K_{i_1,i_1}(\omega))^2 \tag{2}$$

Here $k_{ip,ip_1}(\omega)$ are the correlation functions (defined in the customary way) of the transverse components of the spins. The expression takes its simplest form in the approximation of a self-consistent field for a pure crystal, in which case the correlation functions satisfy $K_{12} = K_{21} = 0$ and $K_{11}(\omega) = -K_{22}(-\omega) = (2S/\omega_e - \omega)$, where $\omega_e = V(0)S$, $V(0)$ is the zeroth Fourier component of V_{ij}, and S is the spin. It turns out that we have

$$\delta F = -N\frac{D^{ex}(0)S^2}{2\omega_e} , \tag{3}$$

where N is the number of spins in the sublattice. The correction to the sublattice magnetization M is then found by differentiating with respect to the molecular field ω_e:

$$\frac{\delta M}{M} = -\frac{D^{ex}(0)}{4SV^2(0)}. \tag{4}$$

Expressions (3) and (4) were derived by perturbation theory and are accordingly valid under the conditions $D^{ex}(0) \ll V^2(0)$ and $D^{1/2}(0) \ll zV(0)$, where z is the average number of spins within the interaction radius. The latter inequality guarantees that there are no local levels. Correction (4) tends toward zero in the classical limit $S \to \infty$, as it should. It is easy to see that this correction remains finite in the Sherington-Kirkpatrick model, which describes a frustrated magnetic material in the approximation of an infinite interaction radius. In a highly disordered antiferromagnet, this model contains no numerical or parametric small quantities. The spatial spin-wave dispersion of the correlation functions causes only a slight increase in the correction, since the contributions from those correlation functions which are diagonal and off-diagonal in terms of the sublattice indices cancel out in the sum of the correlation functions in parentheses in (2) in the small-momentum limit $q \to 0$. This result can be understood easily by recalling that the transverse susceptibility of an antiferromagnet is finite at $q = 0$.

The spatial dispersion is more important if the anisotropy of the interaction fluctuates. Let us assume, for example, that the fluctuating part of the Hamiltonian is

$$\mathcal{H} = -\sum_{i,j} J_{ij}(S_i^x S_j^x + S_i^y S_j^y), \quad \widetilde{J_{ij}} = 0, \quad J_{ij}^2 = D_{ij}^{an}, \tag{5}$$

and leads to a competition between the orientations of the spins along and across the z axis. In this case we have

$$\delta F = -\frac{1}{8\pi}\int d\omega \sum_{i,j} D_{ij}^{an}(K_{i_1,j_2}(\omega)K_{j_2,i_1}(\omega) + K_{i_1,i_1}(\omega)K_{j_2,j_2}(\omega)). \tag{6}$$

The second term in (6) is the most important. After an integration over frequency and a differentiation with respect to ω_e, it leads to the following expression for the correction to the moment of the sublattice:

$$\frac{\delta M}{M} = -\frac{1}{4}D^{an}(0)\,\omega_e^3 S \int \frac{d^d p d^d q\, \Omega_0^2}{(2\pi)^{2d}} \frac{\omega_p^2 + \omega_q^2 + \omega_p^2\omega_q^2/\omega_e^2}{(\omega_p + \omega_q)\omega_p^3\omega_q^3}, \tag{7}$$

where $\omega_p = (2\Delta\omega_e S + S^2(V^2(0) - V^2(p))^{1/2}$ is the frequency of the spin waves in an antiferromagnet with a uniaxial-anisotropy energy Δ, and Ω_0 is the volume of a unit cell.

In the three-dimensional case ($d = 3$), large momenta contribute to one of the integrals in (7), while the second leads to the large logarithmic factor

$$\frac{\delta M}{M} = -\alpha_3 \frac{D^{an}(0)}{SV^2(0)} \ln\frac{V(0)}{\Delta}, \tag{8}$$

where the number α_3 is approximately 1/2 for the model of a nearest-neighbor interaction.

The anisotropy Δ cannot approach zero since this result would violate the stability of the ground state of an easy-axis antiferromagnet.[1] Correction (8) is thus small in crystals which are only slightly disordered. On the other hand, it is logarithmically greater than correction (4), calculated in the molecular-field approximation.

The first term in (6) makes a contribution of the same type as (8), but with a smaller numerical factor.

In the two-dimensional case we have

$$\frac{\delta M}{M} = -\alpha_2 \frac{D^{an}(0)}{SV^2(0)} \left(\frac{V(0)}{\Delta}\right)^{1/2} \ln\frac{V(0)}{\Delta} \ll 1, \quad \alpha_2 \approx 1. \tag{9}$$

We see that in the two-dimensional case the spatial irregularities in the anisotropic interaction are even more substantial than in three dimensions.

The effects discussed here should be seen particularly vividly in a highly disordered antiferromagnet with a spin $S = 1/2$ near the point of the concentration transition to a spin glass. Among such substances are the compounds[2] $La_{1-x}Sr_xCuO_4$ and[3] $YBa_2Cu_3O_{7-\delta}$, whose compositions are similar to those of high-temperature superconductors.

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