Inevitability of the formation of periodic structures in quasi-2*D* planar magnetic materials and type II superconductors

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The transition to the normal state in quasi-2D superconductors may go through an intermediate incommensurate phase consisting of a 1D lattice of normal planes. Arguments are presented in favor of this possibility. Similar conclusions hold for quasi-2D XY magnetic materials.

Phase transitions caused by a spontaneous creation of defects have been discussed in several places in the literature (e.g., Ref. 1). A well-known example is the incommensurate-commensurate phase transition associated with the formation of domain walls (solitons). Friedel[2,3] has recently pointed out that the region in which the superconducting phase exists in quasi-$2D$ superconductors is bounded by the temperature which marks the beginning of the spontaneous creation of vortex loops which are localized between conducting planes. For quasi-$2D$ systems this temperature is substantially lower than the temperature of the $2D$ superconducting transition (which is the same, in order of magnitude, as the temperature of the bulk phase transition, in the mean-field approximation). Similar arguments hold for quasi-$2D$ XY magnetic materials. For definiteness we will restrict the discussion to superconductors.

We will show below that the temperature found by Friedel can be interpreted as the temperature of the spontaneous creation of planes of a normal phase, so we are talking about a commensurate-incommensurate phase transition. We will discuss some anomalies accompanying this transition and also the influence of the discrete nature of the crystal lattice and of the defects on the properties of this phase.

We consider a quasi-$2D$ superconductor with a coupling constant $J_{\parallel}$ in the plane, a coupling constant $J_{\perp}$ between planes, and respective correlation lengths $\xi_{\parallel}$ and $\xi_{\perp}$. In the quasi-$2D$ limit we have $\alpha = J_{\parallel}/J_{\perp} = \xi_{\parallel}/\xi_{\perp} \gg 1$. The temperature $T_c \propto J_{\parallel}/\ln\alpha$ found by Friedel corresponds to the spontaneous creation of a vortex loop with a size on the order of $\xi_{\parallel}$. Since loops of both signs are possible, the interaction between them is of alternating sign, and at $T = T_c$ we can expect the creation of a set of loops, which would form a plane whose properties would be approximately the same as those of the normal phase. The nature of the correlations in the plane cannot be determined more accurately (but this point is not important to the discussion below). As a plane is crossed, the phase of the order parameter changes by π. Planes of this type, localized at $2D$ defects, were discussed previously by Andreev[4] and, even earlier, by Goodman,[5] in connection with a discussion of the possible structure of the mixed state of superconductors. Various planar defects are repelled at large distances, as ordinary domain walls are [in accordance with a law $\exp(-l/\xi_{\perp})$, where l is the distance between solitons], so we would expect the transition to be a second-order phase transition. As in an ordinary commensurate-incommensurate phase transition, anomalies of various types should be seen on the side of the incommensurate phase (see the review by Nattermann and Villain,[6] for example). The distance between solitons will vary in accordance with the ordinary logarithmic law, with corresponding anomalies in the specific heat and other properties.[7] The lower critical field $H_{c_1\parallel}$ vanishes in accordance with $H_{c_1\parallel} \sim T_c - T$, while we have $H_{c_1\perp} \sim (1 - \text{const}\ \xi_{\perp}/l)$; i.e., there is a singularity in the derivative. The angular dependence of H_{c_1} in a system of this sort was actually found by Andreev,[4] who also showed that vortices perpendicular to the planes of the structure may be broken up into parts by normal planes. This circumstance means, in particular, that pinning effects are extremely weakened. Thermal fluctuations in a system of vortices of this sort will be far more pronounced than in an ordinary mixed state, so we could expect the formation of various vortex phases, as predicted by Nelson,[8] with a probability higher than in an ordinary mixed state. As the temperature rises, the density of planes naturally increases, and the distance between planes ultimately becomes comparable in order of magnitude to the width of the planes (the "sinusoidal" regime). A transition to the normal phase follows immediately. Friedel's analysis actually contains an indication that planes are created in certain specific places (between $2D$ conducting planes). This interpretation agrees completely with the circumstance that (as can be shown) the temperature at which a plane undergoes a transition to a rough state is on the order of $J_{\parallel}$; i.e., it is substantially higher than the temperature T_c. The incommensurate phase is thus a sequence of commensurate phases (a devil's staircase).

What influence do defects have? Defects make the domain wall rough at large distances and thus smooth out the effects of the discrete structure. The influence of defects on a wall is described by the relation

$$w \approx AL^{\zeta},$$

where w is a scale value of the transverse displacement of a part of a wall of size L, ζ is a roughness index, and A depends on the nature and concentration (c) of the defects.[6] In the system in which we are interested here, the only possible defects would be of a random-temperature type, for which we would have, quite accurately, $\zeta = 2/5$ (Ref. 6). Using the methods developed in Ref. 6, we can show that the transition temperature is lower than T_c by an amount proportional to $c^{2/5}$, that the distance between solitons varies in accordance with $1/l \sim (T - T_c)^{\beta}$, where $\beta = 1/3$, and that there are obvious changes in the temperature dependences of the specific heat and other properties. Everything said above regarding the behavior of the critical fields remains in force.

It appears that the simplest way to observe the incommensurate phase which we have been discussing here would be to observe a broadening of NMR or nuclear-quadrupole-resonance lines.

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