Operator formalism for fermion strings on Riemann surfaces

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An operator formalism for superghosts of a closed fermion string is proposed. This formalism generalizes the rules for the operator bosonization of $\beta\gamma$ boson systems to the case of Riemann surfaces of arbitrary type. The operator origin of the nonphysical poles is established. Global operator expansions which generate both naive and nonphysical poles are found.

The results of Ref. 1 on multiloop superstring calculations demonstrated just how complex a physical system the fermion string is. Deriving explicit physical results (primarily concerning the vanishing of the cosmological constant) remains a largely unresolved problem. The methods which are used for low types[2] lean heavily on the hyperellipticity and cannot be generalized to higher types. We would thus like to be able to reproduce correlation functions on Riemann surfaces not by a "synthetic" approach as in Ref. 1 but through manipulations which can be carried out explicitly with simple composite elements. The latter can, one can hope, be studied individually. We thus need an operator formalism on Riemann surfaces. With regard to superghost systems (whose contribution is the most important part), the existence of an operator formalism which explicitly generates correlation functions would seem at first glance to be impossible since the correlation functions contain nonphysical poles[1,3] which do not follow from operator products. Our purpose in the present letter is to show that such a formalism can nevertheless be constructed, by a method which generalizes the operator "bosonization of bosons"[4] to arbitrary Riemann surfaces. This bosonization procedure is a fairly subtle construction and one of overriding importance in the local case (in particular, a conserved current corresponding to supersymmetry is constructed in the boson representation). In the local theory of Ref. 4, commuting ghosts γ and conjugate ghosts β are bosonized as

$$\beta = \partial\xi\exp(-\phi), \quad \gamma = \eta\exp\phi, \qquad \xi = \exp\chi, \qquad \eta = \exp(-\chi), \tag{1}$$

where ϕ and χ are two scalar fields; ϕ has the "wrong" sign in front of the two-point correlation function. Scalars cannot be defined unambiguously on Riemann surfaces. Instead of scalars, corresponding currents are the fundamental entities.[5] On the given Riemann surface Σ, of type g, we accordingly introduce two currents, I and ι, with the following two-point correlation functions:

$$\langle 0\,|\,I(u)I(v)\,|\,0\rangle = -\,\omega(u,v), \qquad \langle 0\,|\,\iota(u)\iota(v)\,|\,0\rangle = \omega(u,v). \tag{2}$$

Here $|0\rangle$ is the boson vacuum, with the property $\langle 0|0\rangle = 1$, and ω is the symmetric meromorphic bidifferential[6]

$$\omega(u,v) = d_u d_v \ln E(u,v), \qquad \oint_{a_i} \omega(\cdot, v) = 0, \qquad \oint_{b_i} \omega(\cdot, v) = \omega_i(v), \tag{3}$$

where E is a primform, and ω_i $(i = 1,..., g)$ are holomorphic differentials. We normalize the currents I and ι by the conditions of null a-periods.

Nonvanishing correlation functions on Σ require the presence of $n+1$ fields ξ and n field η (Ref. 1). Let us assume that $x_0 \in \Sigma$ is the position of an "extra" ξ insertion. We also need $2(g-1)$ $\exp\phi$ insertions, which are extras in comparison with the $\exp(-\phi)$ insertions.[1] We denote by z_α $[\alpha = 1,..., 2(g-1)]$ the points of these insertions, while $\mathscr{P} = \Sigma z_\alpha$ is the corresponding divisor. The interaction of I and ι with holomorphic geometry on Σ is described by the following operator "background":

$$B(\mathscr{P}, x_0) = [\theta(\mathscr{P} - 2\Delta - \oint_b I + \oint_b \iota)]^{-1} \exp \frac{-1}{\pi i} \sum_{i=1}^{g} \oint_{a_i} \omega_i(u) \int_{x_0}^{u} I$$

$$\times \left(\exp \int_{2(g-1)x_0}^{\mathscr{P}} I\right) \prod_{\alpha<\beta} E(z_\alpha, z_\beta)^{-1} \prod_\alpha \sigma(z_\alpha)^{-2} \tag{4}$$

(this operator creates from the vacuum $|0\rangle$ a "dressed" vacuum $B|0\rangle$, which corresponds to a $\beta\gamma$ system on Σ). Here Δ is the Riemann class, and σ is a $g/2$ Fay differential.[6] Here and below, an Abel mapping is understood in the arguments of the θ functions. The operator part of the argument consists of the b-periods of the currents I and ι.

All of the constituent operators are assumed to be normally ordered in accordance with

$$:I(u)I(v): = I(u)I(v) + \omega(u,v); \qquad :\iota(u)\iota(v): = \iota(u)\iota(v) - \omega(u,v) \tag{5}$$

and Wick's rule for the senior monoms.

Since background operator (4) incorporates the extra inserts $\xi(x_0)$ and $\Pi^{2g-2}\exp\phi(z_\alpha)$, we are left with the task of writing "neutral" combinations of vertex operators, namely,

$$\widehat{\exp(-\phi(w))\exp\phi(z)} = E(w,z)\left(\frac{E(z,x_0)}{E(w,x_0)}\right)^2 \exp\int_w^z I \tag{6}$$

$$\widehat{\xi(x)\eta(y)} = \frac{1}{E(x,y)} \frac{E(x,x_0)}{E(y,x_0)} \exp\int_y^x \iota \frac{\theta(x + x_0 - 2y - \oint_b j + \oint_b \iota)}{\theta(x_0 - y - \oint_b j + \oint_b \iota)}, \tag{7}$$

where

$$j(u) = I(u) + 2d_u \ln E(x_0, u). \tag{8}$$

The roofs on the left sides of (6) and (7) mean that we are assuming the separate existence of the factors which appear there. On the right side of (6) the product of primforms turns out to be a 1/2-differential in z and a $-3/2$-differential in w, while in (7) it is a 1-differential in y and a 0-differential in x.

The operator-bosonization scheme which we are proposing assumes that the product of any number of inserts (6) and (7) exists against the background of operator B in (4), i.e., that operators (6) and (7) act on the dressed vacuum $B|0\rangle$ ($\langle 0|$ remains the bra-vacuum!).

Merging operators (6) and (7) with the background B into a common, normally ordered expression, we find

$$\prod_{m=1}^{p} :\widehat{\exp(-\phi(w_m))\ \exp\phi(z_m)}: \prod_{l=1}^{n} :\widehat{\xi(x_l)\eta(y_l)}: \; :B:$$

$$= \prod_{l=1}^{n} \frac{\theta(-y_l + \sum_{j=0}^{n} x_j - \sum_{j=1}^{n} y_j + \Sigma qz - 2\Delta - \oint_b I + \oint_b \iota)}{\theta(-x_l + \sum_{j=0}^{n} x_j - \sum_{j=1}^{n} y_j + \Sigma qz - 2\Delta - \oint_b I + \oint_b \iota)}$$

$$\times \frac{1}{\theta(\sum_{j=1}^{n} (x_j - y_j) + \Sigma qz - 2\Delta - \oint_b I + \oint_b \iota)}$$

$$\times \exp\Big(\int_{2(g-1)x_0}^{\mathscr{P}} I + \sum_{m=1}^{p} \int_{w_m}^{z_m} I \Big) \prod_{l=1}^{n} \exp \int_{y_l}^{x_l} \iota \exp \frac{-1}{\pi i}$$

$$\times \sum_{i=1}^{n} \oint_{a_i} \omega_i(u) \int_{x_0}^{u} I \cdot \prod_{0 \leqslant i < j \leqslant n} E(x_i, x_j)$$

$$\times \prod_{1 \leqslant i < j \leqslant n} E(y_i, y_j) \prod_{\substack{i=0 \\ j=1}}^{n} E(x_i, y_j)^{-1} \prod_{k<e} E(z_k, z_e)^{-q_k q_e} \prod_{k} \sigma(z_k)^{-2q_k}, \qquad (9)$$

where $\Sigma q_k z_k = \Sigma z_\alpha + \Sigma_m (z_m - w_m) \equiv \mathscr{P} + \Sigma z - \Sigma w$, and the operator expression on the right side is assumed to be normally ordered.

The correlation functions of the $\beta\gamma$ system on Σ are now reproduced by applying to (9) a $\langle 0|...|0\rangle$ expectation value, which is in turn equivalent to the formal vanishing of all of the operator currents in (9), since expression (9) is normally ordered.

Note the striking changes in the arguments of the θ functions in (9) from those in (4), (6), and (7). These changes stem from the merging of the operator θ functions with the operator exponential functions. The mergings of exponential functions with each other generate numerous c-number factors in (9). It is important to note that the merging of θ functions with each other has no effect at all. The combination $\oint_b \iota - \oint_b I$ is "isotropic" in the sense

$$\langle 0 | (\oint_{b_i} \iota - \oint_{b_i} I)(\oint_{b_j} \iota - \oint_{b_j} I) | 0 \rangle = \tau_{ij} - \tau_{ij} = 0 .$$

The correlation functions are thus indeed generated by the merging of the required number of operator insertions with the operator background. Let us take the insertions separately (under the assumption that they exist against the background of B). From (6) and (7) we find the following expression for the $\beta\gamma$ pair:

$$\widehat{\beta(u)\gamma(v)} = -\partial_u (\widehat{\xi(u)\eta(v)}) \widehat{\exp(-\phi(u)) \exp\phi(v)}$$

$$= -\frac{E(v, x_0)}{E(u, x_0)} \left(\exp \int_v^u \iota\right) \frac{\theta(x_0 - v - \oint_b j + \oint_b \iota)}{\theta(x_0 - u - \oint_b j + \oint_b \iota)} \exp \int_u^v I$$

$$\cdot \left[d_u \ln \frac{E(u, x_0)}{E(u, v)} + \iota(u) + \omega_l(u) \partial_l \ln \theta(x_0 - v - \oint_b j + \oint_b \iota)\right] . \qquad (10)$$

We thus find that the ghost current, defined as

$$j(v) = -\lim_{u \to v} \left(\beta(u)\gamma(v) - \frac{1}{u - v}\right),$$

is the same as expression (8). Correspondingly, for the $\xi\eta$ current $i = \xi\eta$ − (a singularity) we find

$$i(y) = \iota(y) + d_y (\ln E(x_0, y) - \ln\theta(x_0 - y - \oint_b j + \oint_b \iota)) . \qquad (11)$$

This differs from $\iota(y)$ by an operator term which depends on the b-periods of the current j! This term is responsible for the additional nonphysical poles. Specifically, the argument of the θ function in (11) varies as a function of the other operator insertions [each $\exp(-\phi(w))\exp\phi(z)$ shifts it by $z - w$, while $\xi(x)\eta(y)$ shifts it by $x - y$]. We have thus established an operator mechanism by which a current reacts to the positions of the nonphysical poles. Clearly, the nonstandard operator term from (11) is also retained in other expressions involving the current i. For example, we have the merging rules

$$i(x)i(y) = :i(x)i(y): + \omega(x, y)$$

$$+ \omega_i(x)\omega_j(y)\partial_i\partial_j(\ln\theta(x_0 - x + \oint_b \iota - \oint_b j) + \ln\theta(x_0 - y + \oint_b \iota - \oint_b j)) \qquad (12)$$

$$j(u)i(v) = :j(u)i(v): + \omega_i(u)\omega_j(v)\,\partial_i\partial_j \ln\theta(x_0 - v - \oint_b j + \oint_b \iota), \qquad (13)$$

From (10) we also find an energy-momentum tensor:

$$T(v) = -\frac{1}{2}j(v)^2 - \partial_v j(v) + \frac{1}{12}S(v) + \frac{1}{2}i(v)^2 + \frac{1}{2}\partial_v i(v) + \frac{1}{12}S(v) \quad (14)$$

(S is the projective connection on Σ; Ref. 6), and in the operator product TT some additional terms again appear. We will not write out this rather lengthy expression explicitly; it follows directly from the equations written above and from

$$T(u)j(v) = :T(u)j(v): + j(u)\,\omega(u,v) + \partial_u \omega(u,v)\,. \quad (15)$$

One can hope that the "global" operator formalism developed above, which incorporates the global properties of the Riemann surface in the very structure of these operators, will make it possible to make some progress in the analysis of multiloop contributions.[1,3,7] The most direct applications are in constructing at the operator level a supersymmetry current, a BRST charge, and operators for changing the representation on arbitrary Riemann surfaces. A description of strings on Riemann surfaces in terms of the conformal theory thus acquires the same status as in the local case.[4] It would also be interesting to generalize the operations of the joining of Riemann surfaces and the attachment of a handle to the supercase.[8]

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