Bound states of fermions and superconducting ground state in a 2+1 gauge theory with a topological mass term

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An attraction arises between fermions of like charge in a 2 + 1 gauge theory with a topological mass term at certain values of the parameters. The structure of the superconducting state is discussed.

Let us examine a 2 + 1 gauge theory with a topological mass term (a Chern-Simons term). We assume for simplicity that this is an Abelian theory (photons) with one type of fermion (electrons), with the Lagrangian[1,2]

$$L = -\frac{1}{4\gamma} F_{\mu\nu}^2 + \frac{H}{4} \epsilon_{\mu\nu\lambda} F_{\mu\nu} A_\lambda + \bar{\psi}(i\partial\!\!\!/ - A\!\!\!/)\psi - m\bar{\psi}\psi. \tag{1}$$

The topological term may be thought of as being both a seed term and an induced term.[3-5] In this case it arises in the single-loop approximation in an integration over heavy fermions with masses M_i much greater than all of the characteristic momenta in (1) and with

$$H_{ind} = \frac{1}{4\pi} \sum_i M_i/|M_i| = n/4\pi, \quad n = N^+ - N^-. \tag{2}$$

In the non-Abelian case, precisely these values of H are permitted.

1. The presence of a topological term has the consequence that the photon acquires a mass $\mu = \gamma H$, and an additional transverse structure $\sim \epsilon_{\mu\nu\lambda} p_\lambda$ arises in the propagator:

$$G_{\mu\nu}(p) = \frac{\mu}{H} \frac{g_{\mu\nu} - p_\mu p_\nu/p^2}{p^2 - \mu^2} - \frac{i}{H} \frac{\mu^2}{p^2(p^2 - \mu^2)} \epsilon_{\mu\nu\lambda} p_\lambda + \alpha p_\mu p_\nu . \tag{3}$$

This additional structure has an important consequence: A charge which is at rest is a source of not only an electric field but also a magnetic field. The potentials and the fields corresponding to a point charge are (here and below, we are using $e = 1$)

$$A_0 = \Phi = \frac{1}{H} \frac{\mu}{\mathbf{p}^2 + \mu^2}, \quad A_i = \frac{i\mu^2}{H\mathbf{p}^2(\mathbf{p}^2 + \mu^2)} \epsilon_{ij} p_j, \quad i, j = 1,2$$
$$F = \frac{1}{2} \epsilon_{ij} F_{ij} = \frac{\mu^2}{H} \frac{1}{\mathbf{p}^2 + \mu^2} = \mu\Phi \tag{4}$$

in the momentum representation $[p_\mu = (0,\mathbf{p})]$ or

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$$F(\mathbf{r}) = \mu\Phi(\mathbf{r}) = \frac{\mu^2}{2\pi H} K_0(\mu r) \tag{5}$$

in the coordinate representation.

We now consider the interaction of two electrons, which is described in the non-relativistic approximation (justified by the solution which we find) by the Pauli equation for a particle with a reduced mass $m/2$ in external electric and magnetic fields:

$$\left[\frac{(\mathbf{p}-\mathbf{A})^2}{m} + \Phi(\mathbf{r}) - \frac{1}{m}F(\mathbf{r})\right]\Psi = \epsilon\Psi . \tag{6}$$

Substituting (4) and (5) into (6), and transforming to $\Psi(r,\theta) = \Psi_l(r)\exp(il\theta)$ we find

$$-\frac{1}{rm}\frac{d}{dr}\left(r\frac{d\Psi_l}{dr}\right) + \left[\frac{(l-c(r))^2}{mr^2} - \left(\frac{\mu}{m}-1\right)\frac{\mu}{2\pi H}K_0(\mu r)\right]\Psi_l = \epsilon_l\Psi_l \tag{7}$$

$$c(r) = rA_\varphi(r) = \frac{1}{2\pi H}(1-\mu r K_1(\mu r)), \tag{8}$$

where we have $l = 2k+1$ by virtue of the Fermi statistics. If there are several types of fermions, even values of the angular momentum would also be allowed in the pairs of different fermions that form. If $\mu/m > 1$, the magnetic attraction is stronger than the electrical repulsion, and we find the possibility that a bound state will form. If $H \ll 1$, we have the strong-binding case, in which energy levels definitely exist for several low values of l, but in the more interesting case of an induced (or non-Abelian) H [see (2)] there is a weak binding $(m/2)\int U(r)rdr = [(\mu-m)/\mu]n^{-1} < 1$. In the case of a constant angular momentum, a bound state exists only in the s-wave.[6] It can be seen from (7) and (8), however, that the effective moment depends on the distance, so in a certain interval of H values there also exists a bound state in the case in which we are interested, $l=1$ (the p-wave). We assume $H_0 = (2\pi)^{-1}$ [$n=2$; see (2)]. The centrifugal barrier then disappears completely at large distances (this is the known transmutation of spin[7,8]), and we are dealing with an effective s-wave equation with a potential

$$U(r) = \frac{\mu^2}{m}\left[K_1^2(\mu r) - \frac{\mu-m}{\mu}K_0(\mu r)\right], \tag{9}$$

in which a bound state exists. A rough estimate of the binding energy (the derivation of this estimate and a general analysis of the spectrum will be published in a detailed paper) yields $\epsilon \sim \mu^2/m\,\exp(-c(\mu/\mu-m)^2)$, $c\sim 1$, under the condition $\mu - m \ll \mu$. The bound state evidently continues to exist in a certain interval of H values, as long as the centrifugal energy is lower than the binding energy at $H_0 = (2\pi)^{-1}$.

2. We now switch from the two-particle problem to the many-particle problem. We assume that we have a Fermi energy $\epsilon_F{}'$, and we also assume that the bound state is described at absolute zero by a wave function of the BCS type:

$$|\Phi\rangle = \prod_i |\psi_{\mathbf{k}_i}\rangle, \quad |\psi_{\mathbf{k}}\rangle = U_{\mathbf{k}}a^+_{\mathbf{k}}a^+_{-\mathbf{k}}|0\rangle + V_{\mathbf{k}}|0\rangle.$$

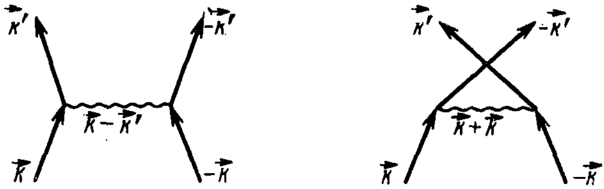

FIG. 1.

We then find a standard equation for the gap, which differs from the usual equation only in the absence of a spin structure:

$$\Delta_{\mathbf{k}} = -\sum_{\mathbf{k}'} V_{\mathbf{k},\mathbf{k}'} \frac{\Delta_{\mathbf{k}'}}{2\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}}, \epsilon_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m} - \epsilon_F, \tag{10}$$

where the potential $V_{\mathbf{k},\mathbf{k}'}$ is determined by the Feynman diagrams in Fig. 1, whose signs are different because of the Fermi statistics (in contrast with the standard case, in which an antisymmetry results from the spin part of the wave function of the pair). After some straightforward calculations we find, for the momentum region of interest here,

$$V_{\mathbf{k},\mathbf{k}'} = -\frac{\mu(\mu-m)}{mH}\left[\frac{1}{(\mathbf{k}-\mathbf{k}')^2+\mu^2} - \frac{1}{(\mathbf{k}+\mathbf{k}')^2+\mu^2}\right] = -\frac{4(\mu-m)}{m\mu^3 H}(\mathbf{k}\mathbf{k}') \tag{11}$$

with $|\mathbf{k}|$, $|\mathbf{k}'| \ll \mu$. The factor $(\mathbf{k}\mathbf{k}')$ reflects the fact that pairing occurs in the p-wave which is similar to that in the case of superfluid He^3 (Ref. 9). Substituting (11) into (10), we find an equation for the gap:

$$\Delta_{\mathbf{k}} = \frac{2(\mu-m)}{m\mu^3 H}\int \frac{d^2\mathbf{k}'}{(2\pi)^2} \frac{(\mathbf{k}\mathbf{k}')\Delta_{\mathbf{k}'}}{\sqrt{\epsilon_{\mathbf{k}'}^2 + |\Delta_{\mathbf{k}'}|^2}}. \tag{12}$$

This equation has the complex solution $\Delta_k = \Delta_0(K_1 + iK_2)$. For Δ_0 we have

$$1 = \frac{2(\mu-m)}{m\mu^3 H}\int \frac{d^2\mathbf{k}}{(2\pi)^2} \frac{k^2}{\sqrt{\Delta_0^2 k^2 + \epsilon_{\mathbf{k}}^2}},\ k^2 = (\mathbf{k}\cdot\mathbf{k}), \tag{13}$$

and in the weak-binding case $\mu - m/\mu \ll 1$ we find

$$\Delta_0 \sim \epsilon_F / k_F \exp(-\pi H\mu^2 / 2(\mu-m)\epsilon_F). \tag{14}$$

Weak binding is natural since the radiative corrections to the mass of the fermion due to the second term in propagator (3) are always on the order of μ/H, so it would be essentially impossible to have $m \ll \mu$.

3. We turn now to the question of a realization of this scenario in real quasi-2D systems. The effective Lagrangian of the 2 + 1 theory is found through reduction from the 3 + 1 theory. The dimensional parameter γ (the charge of the 2 + 1 theory) is

e^2a^{-1} in order of magnitude, where e is the ordinary electric charge, and a is the distance between planes. During the reduction of the free fermion Lagrangian, however, pairs of fermions with masses of opposite sign always appear, and by virtue of (2) we have $H_{\rm ind} = 0$ (Refs. 3–5). This pairing is a consequence of the P and T symmetry of the original model. In order to obtain a P- and T-odd topological mass term we would need to break these symmetries in the original $3+1$ system, e.g., by introducing a magnetic field $\mathscr{H}$ in the direction perpendicular to the reduction plane (or, more generally, by introducing a magnetic order). The electron mass acquires a magnetic increment $\mu\mathscr{H}$, where μ is the magnetic moment, with the same sign for both fermions. Accordingly, $2+1$ fermions with masses $\pm M + \mu\mathscr{H}$ arise, and at large values of $\mathscr{H}$ an $H_{\rm ind}$ arises. The bound state in (7) which was found is also P- and T-odd: It exists only for $l = +1$, not for -1 (see also Ref. 10 regarding the P and T breaking).

It would be interesting to see whether the model proposed here is pertinent to high-temperature superconductivity and how it is related to the models of antiferromagnetic order in CuO ceramics.[11] The results which we have found will be published in detail in a separate paper.

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