Fermion states in an antiferromagnet in connection with a local $U(1)$ symmetry

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The interaction of the fermion degrees of freedom which arise during the doping of an antiferromagnet with magnetic degrees of freedom is analyzed. An invariance group of an antiferromagnet, $G_{\rm AFM}$, is constructed for the purpose. A local $U(1)$ symmetry arises in a natural way for a collinear antiferromagnet in the CP^1 representation for fermions. The local $U(1)$ symmetry and the discrete elements of $G_{\rm AFM}$ are used to construct a long-wave fermion Hamiltonian.

The interaction of fermion degrees of freedom with magnetic degrees of freedom in an antiferromagnet has recently attracted considerable interest. In general, some significant progress has been achieved here, but for the most part on the basis of various elementary models.[1–5] It seems obvious that among this set of elementary models and assumptions regarding the fermions in an antiferromagnet there are some assertions which are of a purely symmetry nature and which are unrelated to elementary assumptions. In this letter we examine the symmetry consequences of the existence of an antiferromagnetic order for additional particles in an antiferromagnet, and we construct a long-wave Hamiltonian for the fermions in an antiferromagnet.

We consider a collinear antiferromagnet characterized by an order parameter: the unit vector $\mathbf{n}$. The antiferromagnet has a local $U(1)$ symmetry, associated with rotations in spin space around $\mathbf{n}$. In a pure antiferromagnet this symmetry is not important, but if fermions are present it becomes explicit and leads to a transformation of the phase of the fermion wave function. The existence of an invariance under a local group leads in an obvious way to a restriction on the form of the fermion Hamiltonian. In addition, we will show that if there is an external magnetic field, or if there is a ferromagnetic vector in the system, the fermions will interact not only with $\mathbf{n}$ but also with the vectors $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$ which are perpendicular to it; here $\mathbf{\Delta}_1 \times \mathbf{\Delta}_2 = \mathbf{n}$ is the triad of unit vectors in spin space. We will use the CP^1 ansatz to illustrate the fermion representation of $G_{\rm AFM}$. We restrict the discussion to a $2D$ square lattice.

We consider an antiferromagnet without additional particles. In a disordered phase, the invariance group consists of the subgroups

$$G = \{D_4, T, R, SU(2)_{spin}\}, \qquad (1)$$

where D_4 is the point group of the crystal, T is the translation group, R is the operation of time reversal, and $SU(2)$ is the rotation group in spin space. Upon a spontaneous breaking of the symmetry, and order parameter arises: the unit vector $\mathbf{n}$, on the sphere $S^2 \approx SU(2)/U(1)$. One of the generators from the $SU(2)$ rotation group—that corresponding to rotations around n—remains unbroken (has no vacuum expectation val-

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ue). This circumstance leads to the local $U(1)$ symmetry in an antiferromagnet.[4,8] The invariance group of the antiferromagnetic state is (we are using a translation cell centered at a site)[6,7]

$$G_{\rm AFM} = (D_4,\ RT,\ T^2,\ U(1)) \subset G. \tag{2}$$

The doubling of the period and the appearance of two sublattices follow from the coupling of T and R in $G_{\rm AFM}$. Under G, $\mathbf{n}$ transforms in accordance with

$$D_4\mathbf{n} = \mathbf{n}, \quad T\mathbf{n} = -\mathbf{n}, \quad R\mathbf{n} = -\mathbf{n}. \tag{3}$$

We now assume that if there is a low concentration of fermions in the system, the antiferromagnetic order described by $G_{\rm AFM}$ is retained in the system. We consider a fermion described by a wave function $\psi(r)$, which has some additional indices which we are not writing out here. This wave function transforms under an irreducible representation of $G_{\rm AFM}$. Since $G_{\rm AFM}$ contains time reversal R, we consider a double-valued irreducible representation of the subgroup (E,RT,T^2) (the point group is not important):

$$\begin{aligned} T^2\psi &= -e^{-2i\lambda}\psi & T^2\chi &= -e^{-2i\lambda}\chi \\ RT\psi &= e^{i\lambda}\chi^* & RT\chi &= -e^{i\lambda}\psi^*, \end{aligned} \tag{4}$$

where $e^{-2i\lambda}$ is the character of the translation group T^2. Representation (4) arises in a natural way when we use the CP^1 formalism. For this purpose, we associate with the order parameter $\mathbf{n}$ a spinor (z_1,z_2): $\mathbf{n} = z^*\sigma z$. We require

$$Rz_\alpha = \epsilon_{\alpha\beta}z^*_\beta \qquad T_\alpha z_\alpha(r) = \epsilon_{\alpha\beta}z^*_\beta(r+a). \tag{5}$$

Relations (3) then follow from (5). Under the assumption that there are two sublattices, A and B, we expand the initial real fermion in the coherent states of these sublattices (an analog of a Bogolyubov transformation):

$$\hat{\psi}_\alpha = \psi(r)z_\alpha(r) + \chi(r)\epsilon_{\alpha\beta}z^*_\beta(r), \qquad T\hat{\psi}_\alpha = e^{-i\lambda}\hat{\psi}_\alpha. \tag{6}$$

In the limit we can assume that the amplitudes ψ and χ vanish on sublattices B nd A, respectively, as was done in Refs. 4 and 9. Since (6) is an expansion of the original real fermion in the spinless particles ψ and χ and the spinors z and ϵz^*, we find from (5) and (6)

$$R\psi = \psi^*, \quad R\chi = \chi^*, \quad T\psi = e^{-i\lambda}\chi, \qquad T\chi = -e^{-i\lambda}\psi. \tag{7}$$

From (7) we finally find double-valued representation (4), which we can postulate. An advantage of using the CP^1 representation is the obvious local $U(1)$ symmetry

$$z \to e^{-i\alpha}z \qquad \psi \to e^{i\alpha}\psi, \quad \chi \to e^{-i\alpha}\chi. \tag{8}$$

Although (8) does follow from elementary considerations, it is, along with (4), a representation of $G_{\rm AFM}$. For use below, we introduce the gauge potential which corresponds to $U(1)$: A_μ, $\mu = x,y$, given by

$$A_\mu = z^* \frac{\partial_\mu}{i} z, \qquad A_\mu \to A_\mu - \partial_\mu \alpha, \qquad z \to e^{-i\alpha} z$$

$$RA_\mu = -A_\mu, \qquad D_\mu = \partial_\mu + iA_\mu . \tag{9}$$

A case of physical interest is that in which there is a ferromagnetic vector $\mathbf{h}$, in addition to the antiferromagnetic vector:

$$R\mathbf{h} = -\mathbf{h}, \qquad T\mathbf{h} = \mathbf{h}, \qquad D_4\mathbf{h} = \mathbf{h} .$$

For example, there could be an external magnetic field or a small ferromagnetic vector corresponding to a local magnetization.[10,3]

It is now a straightforward matter to write a fermion Hamiltonian for the particles in an antiferromagnet, after constructing all possible invariants. We assume that the dynamic variables in the antiferromagnet are the amplitudes ψ and χ, and we restrict the analysis to the long-wave Hamiltonian, assuming a simple minimum of $k^2/2m$ for the fermions at the center of the Brillouin zone. Requiring conservation of $U(1)$ symmetry (8), we can then write the Hamiltonian in the form

$$\begin{aligned}\mathcal{H}_{int} = \int d^2x \{ &\frac{1}{2m} |D_\mu \psi|^2 + \frac{1}{2m} |D_\mu^+ \chi|^2 + \mu(\psi^+\psi + \chi^+\chi) \\ &+ g[(\psi^+\psi - \chi^+\chi)\mathbf{h}\mathbf{n} + \psi^+\chi(\vec{\Delta}_1 + i\vec{\Delta}_2)\mathbf{h}] \\ &+ \tilde{g}[(\psi^+ D_\mu \psi - \chi^+ D_\mu^+ \chi)\mathbf{h}\partial_\mu \mathbf{n} + \psi^+ D_\mu^+ \chi \mathbf{h} D_\mu (\vec{\Delta}_1 + i\vec{\Delta}_2)] + q\,\psi^+ D_\mu^+ \chi z^* \epsilon \partial_\mu z^* + \text{h. c.}\}\end{aligned} \tag{10}$$

We have restricted the expression to terms of second order in the gradients. All the constants in (10) are scalars.

Interestingly, in the Hamiltonian the $U(1)$-noninvariant combination $\psi^+\chi$ appears with the complex vector $\mathbf{\Delta}_1 + i\mathbf{\Delta}_2 \approx z^*\sigma\epsilon z^*$:

$$R(\vec{\Delta}_1 + i\vec{\Delta}_2) = T(\vec{\Delta}_1 + i\vec{\Delta}_2) = (\vec{\Delta}_1 + i\vec{\Delta}_2)^* \tag{11}$$

Here $\mathbf{\Delta}_1$, $\mathbf{\Delta}_2$, and $\mathbf{n}$ form a complete system of vectors in spin space. Under transformation (8), the vectors $\mathbf{\Delta}_1$ and $\mathbf{\Delta}_2$ rotate around $\mathbf{n}$: $\mathbf{\Delta}_1 + i\mathbf{\Delta}_2 \to (\mathbf{\Delta}_1 + i\mathbf{\Delta}_2)e^{2i\alpha}$.

In summary, we have worked from the symmetry of an antiferromagnetic state to construct a Hamiltonian for fermions. The local symmetry in the collinear antiferromagnet has been taken into account explicitly. This symmetry generates a gauge field A_μ, against the background of which the fermions exist. It would be interesting to study the possible existence of nontrivial self-localized solutions in (10). A similar but not identical Hamiltonian was constructed in Ref. 11 on the basis of other principles.

We wish to stress that the double-valued representation which we have constructed explicitly incorporates the local $U(1)$ symmetry, is not exclusive, and can be illustrated in the very simple example of a spin density wave. We also see that in the case considered here the fields ψ and χ have different charges under $U(1)$. It is possible to construct a representation in which ψ and χ would have the same charges under the local group or would not have any charge at all. The case of a collinear antiferromagnet apparently corresponds to only the case discussed by us. It can thus be asserted that an antiferromagnet is a $U(1)$ gauge theory. We have not taken up the specific properties of the point group, which are manifested at special points of the Brillouin zone.[11]

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Self-excited oscillations of the exciton density and temperature in an impurity molecular crystal

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A model in which self-excited oscillations in the exciton-impurity system of a molecular crystal have been observed for the first time is proposed. This model has been tested experimentally.

In the present letter we will analyze a molecular crystal in which there are exciton capture centers. At low exciton densities the quantum yield of the exciton luminescence is approximately equal to unity and the principal mechanism by which energy is transferred from excitons to the lattice is the bimolecular annihilation. Exposure of a

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