Generation of poloidal magnetic fields in jet flows

M. A. Gol'dshtik and V. N. Shtern
Institute of Thermal Physics, Siberian Branch of the Academy of Sciences of the USSR

(Submitted 25 January 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **49**, No. 5, 266–268 (10 March 1989)

An axisymmetric laminar magnetic dynamo can occur in a self-similar jet flow of a viscous liquid if the velocity and the induced magnetic field are inversely proportional to the distance from the origin of coordinates.

According to the well-known results of Cowling and Braginskiĭ, an axisymmetric dynamo is not possible.[1] A condition of the theorem is the assumption that the magnetic field decreases $\sim r^{-3}$ at infinity. In the present letter we examine the case of a slower decay, which is typical of several self-similar jet flows of viscous liquids[2] and electrovortex flows.[3] We seek a solution with fields of the velocity and of the magnetic induction which can be represented as follows in the spherical coordinate system (r,θ,φ):

$$v_\theta = -\nu y(x)(r\sin\theta)^{-1},\quad v_r = -\nu y' r^{-1},\quad v_\varphi = B_\varphi = 0$$

$$B_\theta = -C\phi(x)(r\sin\theta)^{-1},\quad B_r = -C\phi' r^{-1},\quad x = \cos\theta .$$

The prime means differentiation with respect to x, ν is the kinematic viscosity, and C is a constant to be determined. Substituting into the MHD equations, and carrying out some simple manipulations, we find the system of equations

$$(1-x^2)y' + 2xy - y^2/2 = R(1-x^2) - \frac{1}{2}AR^2\,[\phi^2 - \phi_0^2\,(1-x)^2]$$

$$(1-x^2)\phi'' = \beta(y\phi' - y'\phi),\qquad \phi_0 = \phi(0),$$

where R, A, and β are the Reynolds, Alfvén, and Batchelor numbers. From the requirement that the velocity and induction be regular at the symmetry axis we find $y(\pm 1) = \phi(\pm 1) = 0$. There exists a purely hydrodynamic solution (H) with $\phi \equiv 0$. The bifurcation problem is to seek the values $R_m = \beta R$ at which a nontrivial solution arises for ϕ.

Analysis shows that in the case of a Landau jet, with $R = y'(-1)$, $y = 1(1 - x^2)/(4R^{-1} - 1 - x)$, a bifurcation of the magnetic field does not occur at any finite R_m. For a flow described by the Squire solution, however, the result is different[2]: $R = y'(0)$,

$$y = R(1 - x)/\{\chi \operatorname{cotg}[\chi \ln(1 + x)] - 1/2\}, \quad \chi = \frac{1}{2}(2R - 1)^{1/2}.$$

Let us assume that the flow is formed by a converging motion of the material of a plane. In this interpretation, Squire's solution can serve as a very simple model of the jets which are observed near young stars and galactic nuclei.[4]

Calculations show that for laminar motion the excitation of a field is possible only for $\beta \geqslant 0.232$, beginning at point K on curve 1 in Fig. 1. As β increases, the critical value of R_m varies from $R_m^* = 1.74$ at point K to $R_m^* = 3.5$ *as* $\beta \to \infty$ (the dashed line). On line 2 ($R = 7.67$) the jet becomes infinitely strong, and the solution ceases to exist.[2] Curve 3 is the boundary of the region in which the MHD regime (MH) exists. Near this boundary we see the formation of not only a dynamic jet but also a magnetic jet, and on the curve itself v_r and B_r become infinite at $x = 1$. As $\beta \to 0$, curve 3 goes on to the asymptote $R_m = 2.52\beta^{-1/2}$.

The function $\phi(x)$ is symmetric. With the normalization $\phi(0) = 1$ the Alfvén number A is determined from the ratio B_θ / v_r at $x = 0$. The dependence $A(R)$ at $\beta = \text{const}$ (curve 1 in Fig. 2) shows that solution MH arises as a result of a direct fork bifurcation. An asymptotic expression for solution MH as $R \to \infty$ can be derived analytically. Outside a small neighborhood of the equatorial plane the external solution

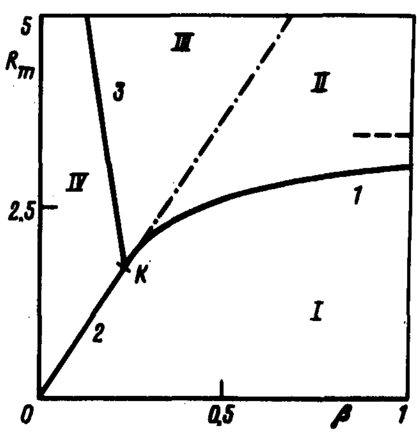

FIG. 1. Map of regimes. In region I there is only a hydrodynamic solution, H; in region II there is, in addition to H, an MHD solution MH; in region III there is only MH; and in region IV there are no laminar solutions.

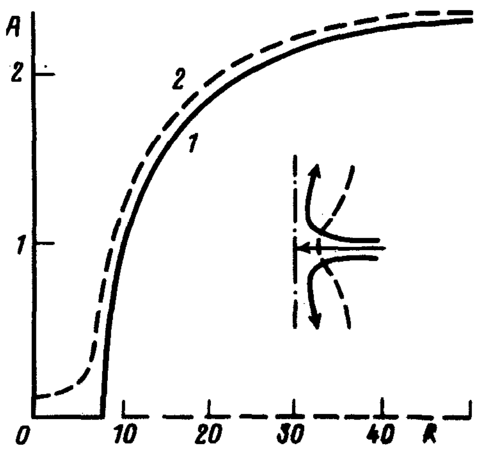

FIG. 2. Intensity of the induced magnetic field versus the Reynolds number ($\beta = 2535$; R_*, $= 7.56$). ($\rightarrow$) Streamlines; ($--$) magnetic field lines.

$\phi_0 = 1 - x$, $y_0 = \beta^{-1}\phi_0$ is valid. Near the plane, a converging fan-shaped jet $y_i = \beta^{-1}[1 - \exp(-R_m x)]$ and a current sheet $j \sim R_m \exp(-R_m x)$ form. The limiting value of the Alfvén number is $A = \beta$. The numerical results shown in Figs. 2 and 3 (curves 2) agree with the asymptotic behavior which has been found.

The self-excitation of magnetic field observed here has a simple physical mechanism. Let us assume that the induction vector of an initial perturbation is directed along the jet axis. We distinguish a liquid conductor with the shape of a torus near the equatorial plane. The flow compresses the torus toward the axis. As a result of the crossing of field lines, an azimuthal electric current arises in it; this current intensifies the magnetic field near the axis. The effect is of the nature of a positive feedback. If the conductivity is low, the intensification of the induction is suppressed by dissipative processes, so an instability develops only at sufficiently high magnetic Reynolds

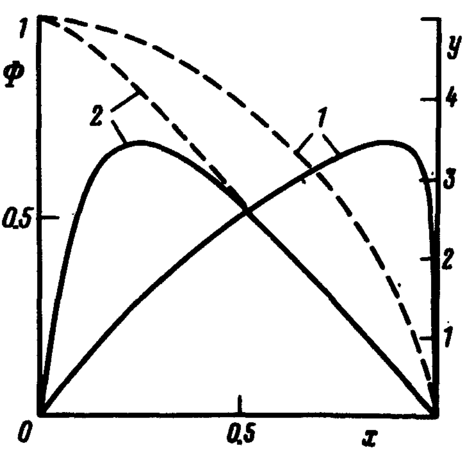

FIG. 3. Distributions of y (solid lines) and ϕ (dashed lines) at (1) the time of bifurcation, $R = R_*$, and (2) at $R = 47.9$ ($\beta = 0.2535$).

numbers. A nonlinear relaxation occurs if a significant fraction of the kinetic energy is converted into magnetic energy, and the flow slows down.

Let us discuss some likely applications. Self-similar solutions usually approach real distributions in a certain geometric region. Let us assume, for example, that the motion occurs in a bounded volume $r_i \leqslant r \leqslant r_0$, $r_0/r_i \gg 1$. There may then exist a region $r_i \ll r_1 < r < r_2 \ll r_0$ in which the flow is approximately the self-similar flow described here, as is typical of jets. If a weak (external) field B_0 is given at the boundary $r = r_0$, the field inside the region at $R_m < R_m^*$ will also be on the order of B_0, while at $R_m > R_m^*$ in the self-similar zone the field will intensify to a level B_s which is given in the limit $B_0 \to 0$ by the solution found here. The fork bifurcation is disrupted if $B_0 \neq 0$, but at $R_m \approx R_m^*$ there is a sharp intensification of the field (curve 2 in Fig. 2). In a region of star formation, the galactic magnetic field would serve as an external field.[5]

The results found here may be taken as evidence that the formation of stars and other massive objects is accompanied by the generation of a poloidal magnetic field. The mechanism observed here for the development of current sheets and magnetic jets may be pertinent to the problem of sunspots.

[1]H. K. Moffatt, Magnetic Field Generation in Electrically Conducting Media, Cambridge U. Press, 1983.
[2]M. A. Gol'dshtik, Vortex Flows, Nauka, Novosibirsk, 1981.
[3]É. V. Shcherbinin (editor), Electrovortex Flows, Zinatne, Riga, 1985.
[4]M. A. Gol'dshtik and V. N. Shtern, Proc. R. Soc. London **A419**, 91 (1988).
[5]A. A. Ruzmaĭkin, D. D. Sokolov, and A. M. Shukurov, Magnetic Fields of Galaxies, Nauka, Moscow, 1988.

Translated by Dave Parsons