The effect of monopoles on the plasma partition function at high temperatures

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The contribution of the monopoles to the plasma partition function at high temperatures is calculated. The monopole density, which is a measure of the nonconservation of the fermion number in the medium, is determined.

We know that grand unified models have monopoles[1,2]: soliton-type particles. At low temperatures monopole fluctuations in a thermodynamic system are unlikely to occur because of the large monopole mass. The partition function in this case is saturated by elementary particles: by gauge- and scalar-field quanta. As the temperature is raised, the probability for monopole fluctuations increases. To determine this probability, let us consider, for simplicity, the gauge theory with a scalar-field triplet from the $SU(2)$ group. In Euclidean space it is described by the Lagrangian

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$$\mathcal{L} = \frac{1}{4}(F^a_{\mu\nu})^2 + \frac{1}{2}(D_\mu \varphi_a)^2 + \frac{\lambda}{4}(\varphi_a^2 - c^2)^2. \tag{1}$$

The monopole solution is[1,2]

$$A^a_i(r) = \epsilon_{ain}\frac{r^n}{r^2} f(r),\ \varphi_a(r) = \frac{r^a}{r} ch(r), \tag{2}$$

$$h(0) = f(0) = 0,\ h(\infty) = f(\infty) = 1.$$

This solution minimizes the static part of the energy functional

$$E(A,\varphi) = \int d^3x \{ \frac{1}{4}(F^a_{ij})^2 + \frac{1}{2}(D_i\varphi_a)^2 + \frac{\lambda}{4}(\varphi_a^2 - c^2)^2 \}, \tag{3}$$

and the monopole mass is

$$M = E_{mon} = \frac{4\pi m_v}{g^2}\epsilon(\lambda/g^2),\quad m_v = gc, \tag{4}$$

where the function ϵ depends only slightly on the argument and $\epsilon(0) = 1$.

Let us assume that the temperature lies in the interval $m_v \ll T \ll c$. The temperature is, on the other hand, low enough to prevent the monopoles from dissolving in the medium, but high enough to satisfy the high-temperature approximation[3]:

$$Z = \mathrm{Tr}\exp(-\frac{1}{T}\hat{H}) \approx N\int dA d\varphi \exp(-\frac{1}{T}E(A,\varphi)), \tag{5}$$

where N is the normalization constant.

Substituting the variables

$$\bar{\varphi} = \frac{\varphi}{\sqrt{T}},\ \bar{A} = \frac{A}{\sqrt{T}},\ \bar{c} = \frac{c}{\sqrt{T}},\ \bar{\lambda} = \lambda T,\ \bar{g}^2 = g^2 T, \tag{6}$$

we reduce the functional integral to the form

$$Z = N\int d\bar{A} d\bar{\varphi} e^{-E(\bar{A},\bar{\varphi})}. \tag{7}$$

At high temperatures the original model thus reduces to an effective three-dimensional model (3), (7) with effective coupling constants $\bar{\lambda}$, $\bar{g}^2$, and $\bar{c}$.

The $3D$ model (7) was studied by Polyakov[4] in connection with the confinement problem. He calculated the functional integral (7) in the approximation of a low-density monopole gas:

$$Z = Z_{per.\,th} \cdot Z_{mon}$$

$$Z_{mon} = \sum_{N^+, N^- = 0}^{\infty} \int \frac{k^{N^+ + N^-}}{N^+! N^-!} e^{-U_N} \prod_{i=1}^{N^+} d^3x_i^+ \prod_{j=1}^{N^-} d^3x_j^-. \tag{8}$$

Here

$$k=\bar{m}_{v}^{3}\left(\frac{\bar{m}_{v}}{\bar{g}^{2}}\right)^{3/2}\cdot f(\bar{\lambda}/\bar{g}^{2})\exp\left\{-\frac{4\pi\bar{m}_{v}}{\bar{g}^{2}}\epsilon(\bar{\lambda}/\bar{g}^{2})\right\},$$

$$\bar{m}_{v}=\bar{g}\bar{c}=m_{v},$$

where f is a calculable function. The function U_N is the interaction energy of N^+ monopoles and N^- antimonopoles. This function can be written in the form[4,5]

$$U_N=\sum_{i<j}\left(\frac{q_i q_j}{R_{ij}}-\frac{e^{-m_H R_{ij}}}{R_{ij}}\right)\frac{4\pi}{\bar{g}^2}.$$

Here $m_H^2=2\lambda c^2$, $R_{ij}=|x_i-x_j|$, and $q_i=\pm 1$ are the monopole (antimonopole) charges.

If the interaction energy is ignored, we can sum the partition function (8)

$$Z_{mon}=\exp(2kV).$$

The concentration of the monopoles and antimonopoles can thus be easily determined:

$$n^{+}=n^{-}=k=m_{v}^{3}\left(\frac{m_{v}}{g^{2}T}\right)^{3/2}fe^{-M/T}. \tag{9}$$

Note the difference between this distribution and Boltzmann's distribution which holds at low temperatures, $T\ll m_v$:

$$n_B^{\pm}=\left(\frac{MT}{2\pi}\right)^{3/2}e^{-M/T}.$$

Note also that Eq. (9) was found from effective theory (7) which ignores the continuum which leads to a renormalization of the coupling constants by the temperature corrections: $C(T)$, $g^2(T)$, and $\lambda(T)$. As a result, the monopole mass becomes a function of the temperature $M(T)$. A correct transition to a high-temperature approximation was demonstrated in Ref. 6 in the particular case of a $2D$ model.

The internal energy of the gas can be calculated from an equation. This energy is

$$u=T^2\frac{dn}{dT}=Mn-\frac{3}{2}nT-nT\frac{dM}{dT}.$$

In contrast with the Boltzmann distribution, we have $u_B=Mn_B+(3/2)n_BT$.

Since a single monopole leads to a strong nonconservation of the fermion number,[7,8] a concentration of monopole pairs (9) can be regarded as a measure of the nonconservation of the fermion number in the medium due to the monopoles.

This effect supplements the sphaleron mechanism for the nonconservation of the fermion number in the medium. This mechanism was studied in Refs. 9–11.

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