Magnetic transitions in a superconducting UPt_3

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The superconducting phases of UPt_3 in a magnetic field and the vortex lattices corresponding to them have been determined by applying the Ginzburg-Landau theory to nontrivial pairing which arises in multidimensional representations of the order parameter.

A phase transition between two different superconducting states has been observed in experimental studies on the absorption of ultrasound in a superconducting UPt_3 in a magnetic field applied parallel to a sixfold symmetry axis.[1,2] This transition occurred when $H \sim 0.6H_{c2}$, i.e., in the presence of a well-developed lattice of vortex filaments (Fig. 1). Such a behavior of the superconducting was explained by Volovik[4] in terms of the phenomenological understanding of the nontrivial pairing.[3] Volovik[4] considered the special case of a transition from a state with a broken hexagonal symmetry in a weak field to a state with a hexagonal lattice of Abrikosov vortices in strong fields. He has not determined, however, all of the possible phases in a magnetic field. There are, moreover, some experimental data showing that a phase with a hexagonal symmetry can occur in a vanishing field (see the review by Fisk *et al.*[5]). Joint[6] showed that a phase transition can occur in this case, although the order of events is reversed: from a hexagonal lattice of vortices in weak fields to a distorted symmetry lattice in strong fields. Joint,[6] however, used the trial-function method which precludes the possibility of making a strong assertion concerning the phase transition. This problem is solved rigorously in this letter.

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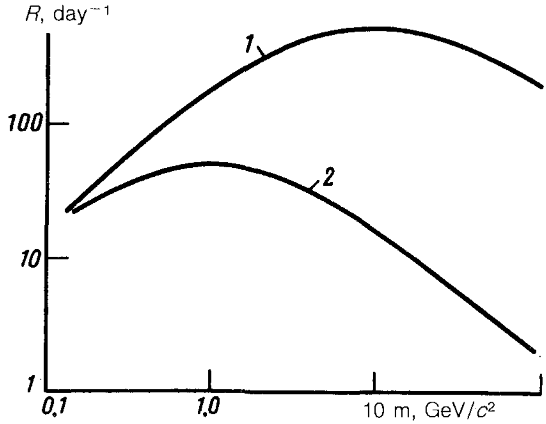

FIG. 3. Phase diagram of UPt_3 versus the applied magnetic field and temperature, taken from Ref. 1. The field is directed along the c axis.

Let us consider the structure of the lattice of vortex filaments and the upper critical field in the region in which the Ginzburg-Landau theory is applicable to the superconducting clusters which arise in a two-dimensional representation E_1 of group D_6, since this representation is the most likely one to describe the set of observable phenomena in UPt_3 below T_c.[5] In this representation the order parameter is a two-component vector $\vec{\eta} = (\eta_x, \eta_y)$. The components of this vector are the coefficients of the expansion of a scalar or vector wave function of the gap, respectively, for a singlet or a triplet pairing, in the basis functions: $\psi(\mathbf{k}) \sim \eta_x k_x k_z + \eta_y k_y k_z$; $\mathbf{d}(\mathbf{k}) \sim \eta_x \hat{z} k_x + \eta_y \hat{z} k_y$ (it is assumed everywhere that $\hat{z}$ is parallel to the c axis). The Ginzburg-Landau functional is invariant with respect to the elements of group D_6. This functional is written in the form[4]

$$F = -\alpha\eta_i^*\eta_i + \beta_1(\eta_i^*\eta_i)^2 + \beta_2|\eta_i\eta_i|^2 + K_1 p_i^*\eta_j^* p_i\eta_j + K_2 p_i^*\eta_i^* p_j\eta_j + K_3 p_i^*\eta_j^* p_j\eta_i, \quad (1)$$

where

$$\mathbf{p} = -i\hbar\vec{\nabla} - \frac{2e}{c}\mathbf{A}. .$$

The solutions corresponding to the homogeneous state in a zero field will have either a hexagonal symmetry

$$\vec{\eta} \sim (1, i) \qquad \beta_2 > 0 \quad (2a)$$

or this symmetry will be disturbed

$$\vec{\eta} \sim (1,0) \text{ or } (0,1) \quad 0 > \beta_2 > -\beta_1. \quad (2b)$$

For the homogeneous state to be stable at $T < T_c$, the fourth-power terms must be positive definite: $\beta_1 > 0$, $B_2 > -B_1$, and the gradient terms must also be positive definite:

$$K_1 + K_2 + K_3 > |K_2|, \qquad K_1 > |K_3|. \quad (3)$$

We introduce in a standard manner the dimensionless units: $\alpha = 1, \beta_1 = 1/2$, and $K_1 = 1$. We will then have $\mathbf{p} = -(i/\kappa)\vec{\nabla} + \mathbf{A}$, where κ is the Ginzburg-Landau parameter. The magnetic field H_{c2}, in which nucleating regions of the superconducting phase appear, is determined from the variation of the free energy while ignoring the terms greater than second power and the screening effects. If we now use $\eta_\pm = \eta_x \pm i\eta_y$ and introduce, with allowance for $[p_x, p_y] = -iH/\kappa$, $a^+ = \sqrt{\kappa/2H}(p_x + ip_y)$, $a = \sqrt{\kappa/2H}(p_x - ip_y)$, and $[aa^+] = 1$, we can write the corresponding Ginzburg-Landau equation in the form

$$\lambda\Psi = \begin{pmatrix} 2(1+C)a^+a + 1 + C - D & 2Ca^+a^+ \\ 2Caa & 2(1+C)a^+a + 1 + C + D \end{pmatrix}\Psi , \tag{4}$$

where Ψ is the column (η_+, η_-), $C = (K_2 + K_3)/2K_1$, $D = (K_2 - K_3)/2K_1$, and $\lambda = \kappa/H$. Solving (4), we introduce the gauge $\mathbf{A} = (-Hy, 0, 0)$ and seek η_+ and η_- in the form of a series in the eigenfunctions of the operators $-i\partial_x, a^+a$—the solutions of the standard problem of the motion of a particle in a magnetic field. All of the possible solutions of (4) are either $(\varphi, 0)$, $(\varphi_1, 0)$ or the family $(p_1\varphi_{n+2}, p_2\varphi_n)$, where φ_n is the wave function n of the Landau level. In the second case there exists an algebraic system for p_1 and p_2 for the eigenvalues of λ. The minimum value of λ corresponds to $H_{c2} = \kappa/\lambda_{\min}$. Using (3), we can show that the minimum of λ is found in one of the two solutions:

$$\Psi \sim (\varphi_0, 0), \quad \lambda_1 = 1 + C - D; \tag{5}$$

$$\Psi \sim (\varphi_2, \frac{\omega}{\sqrt{2}}\varphi_0), \; \lambda_2 = 3(1+C) - (8C^2 + (2 + 2C - D)^2)^{1/2}, \quad \omega = \frac{-4C}{1 + C + D - \lambda_2} . \tag{6}$$

Solution (5) was found by Volovik.[4] In this state we have $\eta_- = 0$, i.e., the order parameter is effectively a single-component parameter, for which an ordinary hexagonal vortex lattice is realized. To effect a transition, Volovik[4] therefore required that the hexagonal symmetry in weak fields be disrupted. We see, however, that at $D < C^2/(1 + C)$ we have $\lambda_2 < \lambda_1$, i.e., at $H \sim H_{c2}$ phase (6) begins to nucleate (see Fig. 2). To analyze the vortices in this phase, we will formulate, following Ref. 7, a periodic solution with respect to the magnetic translations: a lattice of nucleating centers with lattice constants a and b and with an angle α:

$$\eta_+ \sim \sum_n \exp(\pi i \frac{b}{a} \cos\alpha n(n-1)) \exp(\frac{2\pi}{a} inx)(2\kappa H(y - nb\sin\alpha)^2 - 1) \exp(-\frac{\kappa H}{2}(y - nb\sin\alpha)^2)$$

$$\eta_- \sim \sum_n \omega \exp(\pi i \frac{b}{a} \cos\alpha n(n-1)) \exp(\frac{2\pi}{a} inx) \exp(-\frac{\kappa H}{2}(y - nb\sin\alpha)^2),$$

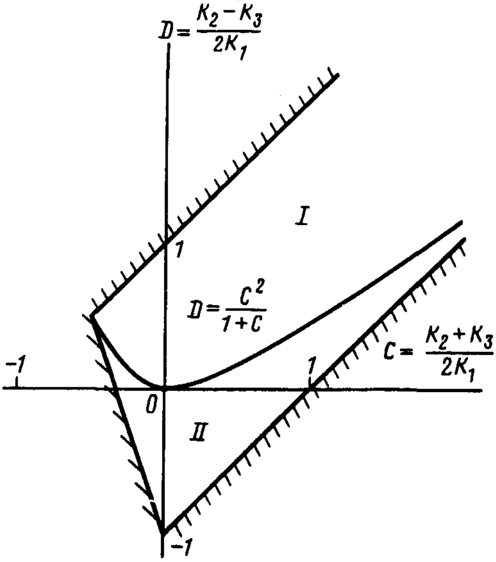

FIG. 2. Phase diagram of UPt_3 versus the dimensionless parameters C and D, determined from the coefficients in the Ginzburg-Landau expansion in a magnetic field $H\sim H_{c2}$. The nucleation centers of phase (5) form in region I and the nucleation centers of phase (6) form in region II. The remaining part of the C and D planes corresponds to parameter values which cannot be realized physically.

where $ab\sin\alpha = 2\pi/\kappa H$ and $\Phi_0 = 2\pi/\kappa$ is the flux quantum in dimensionless units; each cell has a single flux quantum. The lattice constants and the modulus Ψ are determined from the requirement that the energy be invariant under the transformation $|\Psi| \to (1+\epsilon)|\Psi|$, where ϵ is a small number.[7] Imposing this constraint and taking into account that $\kappa \gg 1$ in UPt_3, we find in our gauge that at $H < H_{c2}$

$$F = -\frac{(H_{c_2}-H)^2}{4\delta}, \quad \delta = \frac{\langle F_4 \rangle}{\langle y \dfrac{\delta F}{\delta A_x} \rangle^2}, \tag{7}$$

where F_4 are the fourth-power terms in (1), $\delta F/\delta A_x$ is a functional derivative, and the brackets denote averaging over the volume. The problem thus reduces to a minimization of δ as a function of the lattice constants $\rho + i\sigma = (b/a)e^{i\alpha}$. Calculating the integrals in (7), we find

$$\delta = A_0(C,D,\beta_2)\delta_0(\rho,\sigma) + A_1(C,D,\beta_2)\delta_1(\rho,\sigma) + A_2(C,D,\beta_2)\delta_2(\rho,\sigma)\,. \tag{8}$$

Here A_0, A_1, and A_2 are the functions of the parameters in the Ginzburg-Landau functional (1), and $\delta_0(\rho,\sigma)$ is the standard function of the lattice constants for ordinary superconductors $\delta_0=\sigma^{1/2}\Sigma_{nm}\exp[2\pi i\rho(n^2-m^2)]\exp[-2\pi\sigma(n^2+m^2)]$ (see Ref. 8). The sum is taken over either integer values of n or m or over the half-integer values. The minimum of $\delta_0(\rho,\sigma)$ holds for the hexagonal lattice $\rho=1/2$, $\sigma=\sqrt{3/2}$. The value of δ differs from the value of δ_0 because η_+ and η_- depend on the coordinates in different ways. The structure of the lattice can be most easily determined in the limiting case of large values of $\omega[\omega\to\infty$ as $C\to 0$; see Eq. (6)]. We then find $A_0\sim 1$, $A_1\sim 1/\omega^2$, and $A_2\sim 1/\omega^4$. We therefore retain only the first two terms with

$$\delta_1(\rho,\sigma)=\sigma^{1/2}\sum_{n,m}\exp(2\pi i\rho(n^2-m^2))\exp(-2\pi\sigma(n^2+m^2))(-2\pi\sigma(n^2+m^2)$$

$$+2\pi\sigma nm+16\pi^2\sigma^2(n^4+m^4));\quad \delta'_{1\rho}=0\ .$$

Consequently, δ'_ρ $(1/2, \sqrt{3/2})=0$, but $\delta'_{1\sigma}$ $(1/2, \sqrt{3/2})\neq 0$. Using these results, we immediately find that the lattice is slightly distorted: $\rho=1/2$, $\sigma-\sqrt{3/2}\sim(30/\omega^2)\times(1+2\beta_2)$ (here we have substituted the numerical values of the derivatives for $\rho=1/2$ and $\sigma=\sqrt{3/2}$). For arbitrary values of ω it is generally necessary to take $\delta_2(\rho,\sigma)$ into account, and the distortion is ignored explicitly. If, on the other hand, we take into account that δ_2 depends on ρ by analogy with δ_0 and δ_1, then $\rho=(b/a)\cos\alpha=1/2$ remains, as before, the condition for the minimum of δ, but $\sigma\neq\sqrt{3}/2\,[\delta'_\sigma(1/2,\sqrt{3}/2)\neq 0]$; i.e., the distortion remains. Furthermore, since $\rho=1/2$, the lattice of phase (6) is comprised of isosceles triangles and has the twofold axes as its symmetry elements.

We thus find that at $H\sim H_{c2}$ phase (6) with a distorted vortex lattice may exist. Our experiment can therefore be explained in terms of a phase transition from the symmetric phase to the asymmetric phase. The lattice symmetry of phase (6) is the same, we might add, as that of phase (2b) in weak fields.[4] This similarity confirms in a certain sense the arguments[9] regarding the small phase splitting which was observed in UPt_3 at $H=0$ (Fig. 1). According to Sirgist *et al.*,[9] this splitting occurs because of the presence of antiferromagnetic order in UPt_3 (Ref. 10), which disrupts the hexagonal symmetry of the crystal lattice. As a result, one of the phases, (2*b*), must exist in a narrow region near T_c, and the application of magnetic field at these temperatures does not alter the symmetry and, hence, there is no phase transition in a magnetic field. The type of phase transition capable of producing such a change in the lattice symmetry (from D_{6h} to D_{2h}) can be found from the group analysis.[11] This analysis shows that only a first-order phase transition can occur in this case. An increase in the external magnetic field in this case produces a positive jump in the magnetic induction (magnetization) of the sample.

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