Gluon production in a quasimulti-Regge kinematics

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Gluon production amplitudes are derived in a quasimulti-Regge kinematics. These amplitudes can be used to find corrections to the Regge gluon trajectory, to the vertex for the emission of a gluon from a reggeon, and to the kernel of the equation for the vacuum-channel partial waves.

In the leading log approximation in quantum chromodynamics the t-channel partial waves are expressed in terms of the amplitude for gluon-gluon scattering off the mass shell, which satisfies a Bethe-Salpeter equation.[1,2] This equation can be solved explicitly by virtue of the conformal invariance in the $2D$ impact-parameter space.[3] The "running" nature of the coupling constant was taken into account in Ref. 3 in a calculation of the trajectories of seed pomerons at large momentum transfer, and limitations on their intercepts were given.

To determine the range of applicability of these results, we need to find the corrections to the leading log approximation. In the present letter we derive the amplitudes for inelastic gluon-gluon scattering in the Born approximation for a "quasimulti-Regge kinematics," by which we mean a final-state configuration of the particles such that (1) all of the particles except one pair have large relative energies $\sqrt{s_{ij}}$ and fixed transverse momenta $k_{i\perp}$ and (2) the invariant mass of this particular pair of particles is equal to $k_{i\perp}$ in order of magnitude. We will show that these results make it possible to determine corrections to the gluon Regge trajectory, to the vertex for the emission of a gluon by a reggeon, and to the integral kernel of the Bethe-Salpeter equation.

General expressions for the helicity amplitudes for gluon processes up to six external gluons are given in the literature.[4] Using those general expressions is a fairly complicated matter, however, because of their length and because it is necessary to transform from the helicity basis to the tensor form which is more suitable for our purposes. It turns out to be more convenient to start with dual amplitudes, simplifying them at high energies $\sqrt{s}$ and then taking the field limit $\alpha' \to 0$ (cf. Ref. 5).

The simplest process in the quasimulti-Regge kinematics is the production of an additional gluon in the fragmentation region of the initial gluon (see Fig. 1, where all the notation is explained):

$$s \equiv -2k_0p \sim 2k_1p \sim 2k_2p; \quad -t \equiv 2pp' \sim -2k_0k_1 \sim -2k_0k_2 \ll s. \tag{1}$$

The s-channel helicity of the target particle is conserved here, and the amplitude depends on only its color spin. We have accordingly omitted its spin indices. The amplitude for the process is

$$A^{\mu_0\mu_1\mu_2}_{a_0a_1a_2aa'} = \frac{4g^3}{t} T^b_{a'a} \sum_{\mathcal{P}} T^c_{a_0a_2} T^c_{a_1b} \Delta^{\mu_0\mu_1}(k_0, k_1, p) D^{\mu_2}(k_0, k_1, k_2). \tag{2}$$

Here T_i are the generators of the color group in the associated representation; the summation is over all permutations of the indices (0,1,2);

$$\Delta^{\mu\nu}(k, q, p) = g^{\mu\nu} - \frac{q^\mu p^\nu}{(qp)} - \frac{p^\mu k^\nu}{(kp)} + (kq)\frac{p^\mu p^\nu}{(kp)(qp)},$$

$$D(k_0, k_1, k_2) = \frac{1}{(k_0, k_1)}[(k_1k_2 + \frac{t}{2}\frac{k_1p}{k_2p})p + \frac{k_1p}{k_0k_2}(k_1k_2 - \frac{t}{2})k_0 - (p(k_1+k_2))k_1]. \tag{3}$$

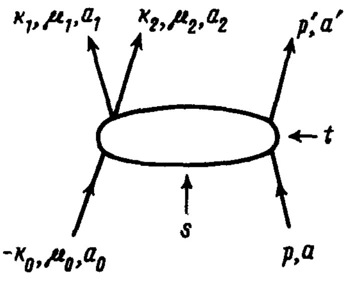

FIG. 1

It is obvious from (2) and (3) that the amplitude $A^{2\to 3}$ is invariant under a gradient transformation of any of the polarization vectors, $e(k_i)\to e(k_i)+ck_i$, regardless of the values of the other vectors. We wish to stress that the amplitude does not have simultaneous poles in overlapping channels here.

As the transverse momentum of one of the gluons which are produced tends toward zero or infinity, the amplitude $A^{2\to 3}$ is inversely proportional to the modulus of this momentum. The effect is a logarithmic behavior of the total cross section. In the multi-Regge limit, expression (2) for the amplitude becomes the well-known factorized expression.[1]

Let us consider the production of two additional gluons in a gluon scattering (Fig. 2) in the kinematic region

$$s \equiv 2p_1p_2 \gg s_{ij} \equiv 2p_ik_j \gg \kappa \equiv 2k_1k_2 \sim -t_i \equiv 2p_ip_{i'} \sim -t \equiv -(p_i - p_{i'} - k_i)^2;$$
$$s_{1i}s_{2j} \sim \kappa s; \qquad i,j = 1,2. \tag{4}$$

In this case the helicities of both of the scattered particles are conserved, and the amplitude depends on only their color spins. This amplitude can be written in the form

$$A^{\mu_1\mu_2}_{a_1a_2aa'bb'} = 2sg^4\,\frac{T^i_{a'a}}{t_1}\left[T^{a_1}_{ij}\,T^{a_2}_{jk}\,A^{\mu_1\mu_2} + \begin{pmatrix} k_1 \leftrightarrow k_2 \\ \mu_1 \leftrightarrow \mu_2 \\ a_1 \leftrightarrow a_2 \end{pmatrix}\right]\frac{T^k_{b'b}}{t_2}, \tag{5}$$

where the tensor $A^{\mu_1\mu_2}$ is orthogonal with respect to the vectors k_1 and k_2, respectively, so we have the same gradient invariance for the amplitude $A^{2\to 4}$ as for $A^{2\to 3}$, as in an Abelian theory. To make this invariance explicit, we introduce the following vectors, which are orthogonal with respect to k_i:

$$a_i = -2\left[q_i + \left(\frac{s_{ji}}{s} + \frac{t_i}{s_{ii}}\right)p_i - \frac{s_{ii}}{s}p_j - \frac{t}{\kappa}k_j\right],$$
$$b_i = 2\left(p_j - \frac{s_{ji}}{\kappa}k_j\right), \quad c_i = 2\left(p_i - \frac{s_{ii}}{\kappa}k_j\right); \quad i,j = 1,2; \; i \neq j. \tag{6}$$

Here

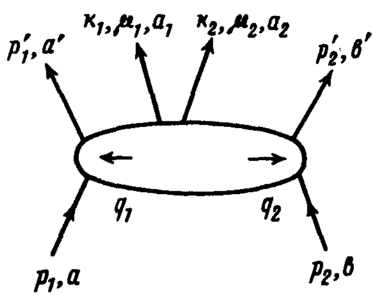

FIG. 2.

$$A^{\mu\nu} = \frac{-a_1^\mu a_2^\nu}{t} + \frac{b_1^\mu b_2^\nu}{s}\left(1 + \frac{s_{11}s_{22}}{st}\right) + \frac{b_1^\mu c_2^\nu}{s}\left(\tau_2 \frac{s}{s_{22}} - \frac{s_{11}s_{12}}{st}\right)$$
$$+ \frac{c_1^\mu b_2^\nu}{s}\left(\tau_1 \frac{s}{s_{11}} - \frac{s_{22}s_{21}}{st}\right) - \left(\frac{c_1^\mu c_2^\nu}{s} + h^{\mu\nu}\frac{t}{\kappa}\right)\left(1 + \frac{\kappa}{t} - \frac{s_{12}s_{21}}{st}\right) \tag{7}$$
$$- h^{\mu\nu}\left(\frac{s_{11}s_{22}}{\kappa s} - \tau_1\frac{s_{12}}{s} - \tau_2\frac{s_{21}}{s} + \frac{s_{11}s_{22}}{st}\right),$$

where

$$h^{\mu\nu} \equiv 2\left(g^{\mu\nu} - \frac{2k_2^\mu k_1^\nu}{\kappa}\right), \qquad \tau_i \equiv \frac{t_i}{(s_{i1} + s_{i2})} . \tag{8}$$

At first glance, it would appear that amplitude (5) contains simultaneous poles in overlapping channels and also second-order poles. However, they cancel out in the sum, and we are left with only the singularities corresponding to the Feynman diagrams. In the multi-Regge limit, amplitude (5) can be factorized, and it becomes the known expression.[1] At small or large $|q_{i\perp}|$ we have the behavior $A^{2\to4} \sim |q_{i\perp}|^{-1}$ for the amplitude, so that the total cross section is logarithmic. The residues of amplitude (5) at the poles $t_i = 0$ are expressed in terms of amplitude (2):

$$t_i A^{\mu_1\mu_2}_{a_1a_2b_1b_1'b_2b_2'}\Big|_{t_i=0} = 2gT^{a_0}_{b_ib_i'}(p_j)_{\mu_0} A^{\mu_0\mu_1\mu_2}_{a_1a_2b_jb_j'}\left(\begin{matrix} k_0 \to q_j \\ p \to p_j \end{matrix}\right), \tag{9}$$

where $i, j = 1, 2;\ i \neq j$.

The amplitudes derived here must be known in order to calculate corrections to the leading log approximation. Specifically, we consider the correction terms ($\sim g^4$) in the equation of the leading log approximation[1] which are shown schematically in Fig. 3. The first term on the right is the correction to the Regge gluon trajectory. To find the three-particle contribution to its imaginary part, we need to know $A^{2\to3}$ in the quasimulti-Regge kinematics (the corresponding two-particle contribution is found with the help of the t-channel unitarity condition through an iteration of the Born amplitude for gluon-gluon scattering). The correction to the effective vertex for the emission of a gluon by a reggeon (the second term) can be calculated from the t_1- and t_2-channel unitarity conditions, again through the use of $A^{2\to3}$. Finally, the correction to the integral kernel of the equation contains a product of two tensors $A^{\mu\nu}$, in terms of which the amplitude $A^{2\to4}$ is expressed (the third term). We hope to publish the results of calculations of these corrections in the near future.

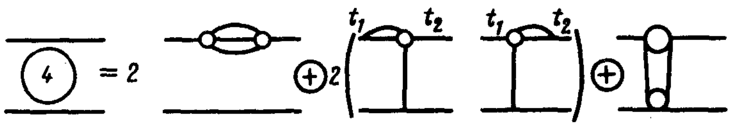

FIG. 3.

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