Dynamic quantization method in the example of 2*D* chiral gauge model

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A new quantization method is proposed for field theories of a wide class. The method is demonstrated in the example of the non-Abelian chiral Schwinger model.

1. I have recently suggested[1] a new method of dynamic quantization, which is implemented in the present letter in a mathematically rigorous form in the example of one of the simplest so-called anomalous gauge theories: the non-Abelian chiral Schwinger model. As a result, this model turns out to be gauge- and Lorentz-invariant with a spectrum of elementary excitations $\omega(k) = k, k > 0$.

We begin with a brief summary of the dynamic quantization method.

In the dynamic quantization, a regularization is carried out by imposing secondary constraints in the deep-UV region. The imposition of secondary constraints is taken into account automatically when corresponding Dirac commutation relations[2] are used in place of the original field commutation relations. The basis for this regularization in the case of asymptotically free theories is that because of the weak interaction, there is the possibility of correctly determining the occupation numbers a^+a of definite modes in the deep-UV region. These occupation numbers are adiabatic invariants of the motion. Furthermore, the corresponding creation (a^+) and annihilation (a) operators commute with the primary constraints, which are generators of gauge transformations. Imposing secondary constraints $a^+ \approx 0$, $a \approx 0$ is thus dynamically self-consistent and does not violate the conventional understanding of gauge theories.

The Fadeev-Shatashvili program for quantizing anomalous theories[3] does go beyond the scope of the conventional understanding of gauge theories.

2. We consider a 1*D* dynamic system with the Hamiltonian

$$H = \int dx \left\{ \frac{1}{2} e^2 (E^a)^2 - i\varphi^+ \nabla_1 \varphi - A_0^a \chi^a \right\}. \tag{1}$$

Everywhere in this paper we are using $\nabla_\mu = \partial_\mu - iA_\mu$, where $\mu = 0, 1$; and $A_\mu = A_\mu^a t^a$, where t^a are the generators of the Lie algebra of the gauge group; φ is a complex Grassmann field; and the fields A_0^a play the role of Lagrange multipliers for the constraints

$$\chi^a = (\nabla_1 E)^a + \varphi^+ t^a \varphi \approx 0 . \tag{2}$$

We will use x, y to represent the spatial coordinate x^1, and we will use t to represent the time. Hamiltonian (1) and the initial commutation relations

$$[\varphi(t,x), \varphi^+(t,y)] = i\delta(x-y)$$
$$[A_1^a(t,x), E^b(t,y)] = i\delta^{ab}\delta(x-y) \tag{3}$$

lead to Heisenberg equations of motion, of which the only relevant one is the Weyl equation

$$i(\nabla_0 + \nabla_1)\varphi = 0. \tag{4}$$

We denote by $\{\varphi_{km}(t,x)\}$, $-\infty < k < +\infty$, $m = 1,\ldots, N$ a complete orthonormal set of solutions of Eq. (4) with the initial conditions (at $t = t_0$)

$$\varphi_{km}(t_0, x) = (P\exp i \int_{-\infty}^{x} A_1(t_0, y)dy)u_m \exp(ikx), \tag{5}$$

where the operator P corresponds to ordering along the integration contour, and $\{u_m\}$ is an orthonormal basis in the space in which the matrices t^a act. We expand the fields φ and φ^+ in sets of functions $\{\varphi_{kn}\}$ and $\{\varphi_{kn}^+\}$ with coefficients $\{a_{kn}\}$ and $\{a_{kn}^+\}$, respectively:

$$\varphi(t,x) = \sum_n \int \frac{dk}{2\pi} \varphi_{kn}(t,x) a_{kn},$$

$$\varphi^+(t,x) = \sum_n \int \frac{dk}{2\pi} a_{kn}^+ \varphi_{kn}^+(t,x).$$

It follows from the definition of the functions φ_{kn}, Eq. (4), and commutation relations (3) that the equations of motion and the vanishing commutation relations in terms of the variables $\{a_{kn}, a_{kn}^+\}$ take the form

$$\dot{a}_{kn} = 0, \quad \dot{a}_{kn}^+ = 0 \tag{6}$$

$$[a_{km}, a_{pn}^+] = 2\pi\delta_{mn}\delta(k-p). \tag{7}$$

We also find the commutation relations

$$[a_{km}, \chi^a] = 0, \quad [a_{km}^+, \chi^a] = 0, \tag{8}$$

and we verify that the variables $\{\chi a_{km}, a_{km}^+\}$ commute with the field A_1 and also that we have the commutation relation $[\varphi_{km}(y), X^a(x)] = t^a\delta(x-y)\varphi_{km}(y)$.

Equations of motion (6) and commutation relations (8) thus show that for a regularization of the theory it is natural to impose the secondary constraints

$$a_{km}^+ \approx 0, \quad a_{km} \approx 0 \ \text{for} \ |k| > \Lambda \to \infty. \tag{9}$$

3. We can now write Dirac commutation relations corresponding to constraints (9). We follow the approach proposed by Dirac[2] in the case of classical mechanics. In going over to quantum mechanics, we must resolve the problem concerning the ordering of operators which arises in such a way that the following relation holds:

$$[A, BC]^* = [A, B]^* C + B[A, C]^*(-1)^{\alpha(A)\alpha(B)}, \quad (10)$$

where $[...,...]^*$ means a Dirac commutation relation, and α is a parity function which is defined on uniform operators with values in Z_2. If the Dirac commutation relations are defined on the fundamental fields $(A_1, E, \varphi^+, \varphi)$ they can be extended by induction to any functionals of fundamental fields with the help of relation (10). If the Dirac commutation relations on the fundamental fields are bilinear, if they satisfy the Jacobi superidentity, and if they satisfy the relation

$$[A, B]^* = -[B, A]^*(-1)^{\alpha(A)\alpha(B)},$$

then all these properties will also be exhibited by Dirac commutation relations for operators of a general type.

Here are the nonzero Dirac commutation relations for the fundamental fields. In calculating these fields we made use of commutation relations (3), (7), and (8):

$$[\varphi(x), \varphi^+(y)]^* = \sum_m \int_{-\Lambda}^{\Lambda} \frac{dk}{2\pi} \varphi_{km}(x)\varphi^+_{km}(y); \quad (11)$$

$$[A^a_1(x), E^b(y)]^* = i\delta^{ab}\delta(x-y),$$

$$[\nabla_1 E^a(x), \varphi(y)]^* = -\sum_m \int_{|p|>\Lambda} \frac{dp}{2\pi} \varphi_{pm}(y)(\varphi^+_{pm}(x)t^a\varphi(x)),$$

$$[\nabla_1 E^a(x), \nabla_1 E^b(y)]^* = if_{abc}(\nabla_1 E^c(x))\delta(x-y)$$

$$+ \frac{1}{2}\sum_m \int_{|p|>\Lambda} \frac{dp}{2\pi} \{[(\varphi^+_{pm}(y)t^b\varphi(y))(\varphi^+(x)t^a\varphi_{pm}(x))$$

$$+ (\varphi^+(y)t^b\varphi_{pm}(y))(\varphi^+_{pm}(x)t^a\varphi(x)) - [(xa)\leftrightarrow(yb)]\}.$$

A direct check easily shows that commutation relations (11) satisfy the Jacobi superidentity. From (8) we also find

$$[\chi^a(x), \chi^b(y)]^* = if_{abc}\delta(x-y)\chi^c(x). \quad (12)$$

When we use Dirac commutation relations, we can assume that constraints (9) hold in the strong sense and that new equations of motion can be found from the formal equations, (4), simply by deleting variables (9). The fermion current $J_{\mu a} = (\varphi^+ t^a \varphi, \varphi^+ t^a \varphi)$ is thus regularized, and we can use Eqs. (4) directly in analyzing its dynamics. In this manner we find

$$\nabla_\mu J^\mu = 0. \quad (13)$$

Equations (12) and (13) mean that the theory is gauge-invariant.

4. The ground state is defined in accordance with $|0\rangle = \Pi_{-\Lambda<k<0} a^+_{k1} ... a^+_{kN}$. The operators which annihilate a meson and a baryon with a momentum $k > 0$ are given by

$$c_k = \int dx \exp(-ikx)\varphi^+(x)\varphi(x), \quad k>0,$$

$$b_k = \int dx \exp(-ikx)\varphi_1(x)\varphi_2(x)\ldots\varphi_N(x), \quad k>0, \tag{14}$$

respectively, where $c_k|0\rangle = b_k|0\rangle = 0$. Since we have $[c_k^+, H]^* = -kc_k^+$ and $[b_k^+, H]^* = -kb_k^+$ states $c_k^+|0\rangle$ and $b_k^+|0\rangle$ have an energy $\omega(k) = k$. Lorentz invariance thus prevails at all momenta if we take the limit $\Lambda \to \infty$. We would also like to call attention to the formula $[c_k, c_p^+]^* = kN\delta(k-p)$, from which it follows that the state $c_k^+|0\rangle$ is normalizable and that the mesons are Bose particles. Gauge-invariant but nonlocal "particle" creation operators do not diagonalize Hamiltonian (1) against the background of ground state $|0\rangle$.

The theory is obviously unitary. Gribov[4] has examined the gauge anomaly from a similar standpoint.

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[3]L. D. Faddeev and S. L. Shatashvili, Teor, Mat. Fiz. **60**, 206 (1984); L. D. Faddeev and S. L. Shatashvili, Phys. Lett. **B167**, 225 (1986).

[4]V. N. Gribov, Preprint KFKI-66, Budapest, 1981.

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