Properties of lattices with "large" Josephson junctions between superconducting granules

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A model of a Josephson lattice which incorporates the self-magnetic fields of the Josephson currents is described. The field H_{c1} and the vortex pinning energy are calculated for the case in which the linear dimension of the superconducting granules is greater than the London length and greater than any length compared with the Josephson length δ.

The unusual properties of high-temperature superconducting ceramics have stimulated research on granular superconductors with Josephson junctions between granules. While the standard approach to the description of such systems starts from the XY model,[1,2] which is applicable in the case of granules with linear dimensions L shorter than the London length λ_L, in the present letter we are reporting results of a study of a lattice of Josephson junctions for the case $L \gg \lambda_L$.

In a lattice of this sort, far from the points at which junctions intersect (over distances greater than λ_L), the phase difference $\theta(u)$ $(u = x,y)$ is described by the equation $\sin\theta = \delta^2\partial^2\theta/\partial u^2$. Here it is necessary to formulate boundary conditions at the sites of the lattice. For convenience we introduce around each site eight quantities: the limits of the functions $\theta(u)$ as site (m,n) is approached from four directions ($\theta_{mn}^{(i)}$, $i = 1,2,3,4$) and its derivatives ($\theta_{mn}^{(i)\prime}$) (Fig. 1). First, from the continuity of the magnetic field we have $H_z = H = (\phi_0/2\pi d)\partial\theta/\partial u$, where $d = 2\lambda_L + d'$; second, by virtue of the continuity of the phase (φ) of the superconducting wave functions within the superconducting granules we have

$$\theta_{mn}^{(1)\prime} = \theta_{mn}^{(2)\prime} = \theta_{mn}^{(3)\prime} = \theta_{mn}^{(4)\prime} = \theta_{mn}^{\prime}, \tag{1a}$$

$$\theta_{mn}^{(1)} + \theta_{mn}^{(2)} = \theta_{mn}^{(3)} + \theta_{mn}^{(4)}. \tag{1b}$$

The sine-Gordon equation and Eqs. (1a) and (1b), supplemented with conditions at the boundary of the lattice with vacuum, completely determine the behavior of the system of Josephson junctions. The joining conditions make no assumption that θ is continuous at the sites, as can be seen quite clearly from the very simple example of a one-vortex solution at two intersecting junctions (Fig. 2). Each branch of the solutions in this figure is part of a one-soliton solution. The energy of such a vortex is $E = (2-\sqrt{2})E_J = \sqrt{2}E_p$, where E_p is the energy of its pinning at an intersection, and $E_j = (4\hbar j_c\delta)/e$ is the energy of a vortex at a linear junction. The corresponding lower critical field is $H_{c1}^* = (2-\sqrt{2})H_{c1}$, where $H_{c1} = 4\pi E_J/\phi_0$ is the critical field for a linear junction. Interestingly, a pinning of this type leads to a resonant absorption of

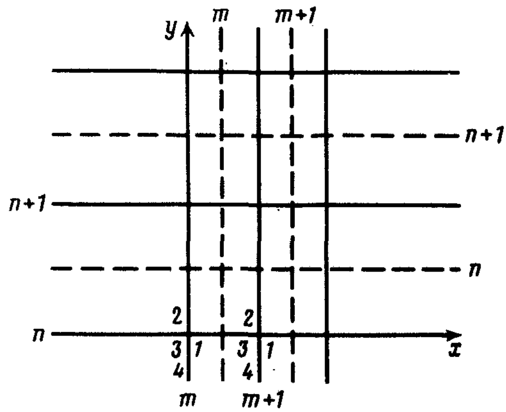

FIG. 1.

electromagnetic radiation at the frequency (here we are ignoring the damping due to normal currents) $\omega = 2^{-3/2}(\sqrt{5} - 1)c_0/\delta$, where $c_0 = c(d'/\epsilon d)^{1/2}$ is the velocity of Swiehart waves.

We consider the case of a "dense" lattice with constants $\lambda_L \ll L_x, L_y \ll \delta$. Under the conditions that the magnetic field is weak and varies slowly from site to site (i.e., $\theta'^2_{m+1,n} - \theta'^2_{m,n} \ll \theta'^2_{mn}$), we find the following energy functional from the sine-Gordon equation and Eq. (1):

$$\mathcal{F} = L_x L_y \sum_{m,n} \{ (\frac{\hbar}{2e} j_c)[\frac{1}{L_x} + \frac{1}{L_y} - \frac{1}{L_x}\cos(\varphi_{mn} - \varphi_{m-1,n} - \frac{2\pi}{\phi_0} L_x A_{x,mn}) - \frac{1}{L_y}\cos(\varphi_{mn} - \varphi_{m,n-1} - \frac{2\pi}{\phi_0} L_y A_{y,mn})] + \frac{1}{8\pi\mu} \hbar^2_{mn} \} . \quad (2)$$

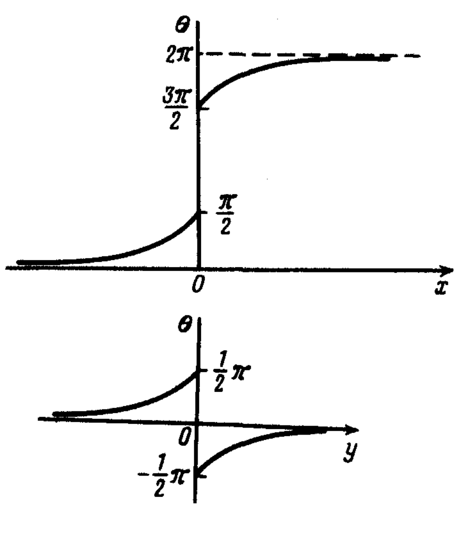

FIG. 2.

Here we have introduced the phases φ_{mn} and the "vector potential" $\mathbf{A}_{mn}$ at site (m,n) of the dual lattice (the dashed lines in Fig. 1):

$$\frac{1}{2}(\theta^{(4)}_{m,n+1} + \theta^{(2)}_{mn}) \equiv \varphi_{mn} - \varphi_{m-1,n} - \frac{2\pi}{\phi_0} L_x A_{x,mn} \quad ,$$

$$\frac{1}{2}(\theta^{(3)}_{m+1,n} + \theta^{(1)}_{mn}) \equiv \varphi_{m,n-1} - \varphi_{mn} + \frac{2\pi}{\phi_0} L_y A_{y,mn} \; .$$

The vector potential $\mathbf{A}$ is related to the effective magnetic field h and to the field at the junction, H:

$$h_{mn} = \mu H_{mn} = \frac{1}{L_x}(A_{y,mn} - A_{y,m-1,n}) - \frac{1}{L_y}(A_{x,mn} - A_{x,m,n-1}).$$

The quantity $\mu = d(L_x + L_y)/L_x L_y$ is an effective magnetic permeability. In the continuum limit, and for a slow variation of the phase, functional (2) takes the form of the anisotropic Ginzburg-Landau functional with a constant order parameter. The effective depths to which the magnetic field penetrates along the x and y directions are $\delta^*_{x(y)} = \delta(L_{x(y)}/(L_x + L_y))^{1/2}$.

Let us describe the structure of a vortex in a dense square lattice $(L_x = L_y = L)$. It is easy to show that at the core of a vortex the contribution of the magnetic field to functional (2) is small. The equation for the phase φ thus takes the same form as in the XY model.[3] Far from the center of a vortex, according to (2), the field satisfies the London equation with an effective penetration depth $\delta^* = \delta/\sqrt{2}$. Joining the solutions for the core and for large r, we find, for both regions,

$$h_{mn} = \frac{\phi_0}{\pi\delta^2} \ln \frac{\delta^*}{L\sqrt{n^2 + m^2}} \; ; \qquad m,\, n > 1 \tag{3a}$$

$$h(r) = \frac{\phi_0}{\pi\delta^2} K_0(r/\delta^*) \quad . \tag{3b}$$

In the general anisotropic case, the vortex energy is

$$E = \frac{\pi\hbar}{2e} j_c \sqrt{L_x L_y} \left(\ln \frac{\delta}{\sqrt{L_x L_y}} + \text{const}\right),$$

and with $L_x = L_y$ this expression is the same as that of Ref. 4. Since the structure of the cores of the vortices in a dense lattice is the same as the structure of the vortices in the XY model, the pinning energies in these systems are also the same: $E_p \approx 0.1\hbar j_c L/e$ (Ref. 3).

Following a procedure similar to that described above, we can also analyze a highly anisotropic lattice: $\lambda_L \ll L_x \ll \delta \ll L_y$. This problem reduces to the Frenkel'-Kontorova model.[5] The energies and critical fields are

$$E/E_J = H^*_{c1}/H_{c1} = (L_x/2\delta)^{1/2}, \qquad E_p = 4\pi^2 E_J \exp(-\pi^2(\delta/2L_x)^{1/2}).$$

The effective penetration depths are $\delta^*_x = (L_x\delta/2)^{1/2}$, $\delta^*_y \sim \delta$. In a system of this sort, the maximum total critical current along the y axis, with $H=0$, is thus $I^*_y \sim j_c\delta^*_x \ll j_c\delta$ (Ref. 6).

Using these expressions, we can find the temperature dependence of E, H^*_{c1}, and E_p for the case of a Josephson junction with an insulating interlayer ($j_c \sim \tau$, $\delta \sim \tau^{-1/4}$, $\tau \equiv (T_c - T)/T_c$). With $L \gg \delta$, we have E, H^*_{c1}, $E_p \sim \tau^{3/4}$. A behavior of this sort for the characteristic magnetic field has been observed[7] in the ceramic $Y_1Ba_2Cu_3O_x$. In the case $\delta \gg L$ we find $E, H^*_{c1}, E_p \sim \tau$; and in the case $L_y \gg \delta \gg L_x$ we find $E, H^*_{c1} \sim \tau^{7/8}$, $E_p \sim \tau \exp(-a\tau^{-1/8})$.

The next interesting limit is the case of strong magnetic fields, in which the field at the junctions is equal to the external field H. In this case the phase difference is a linear function of the coordinates $[\theta(u) = \theta + 2\pi dHu/\phi_0,\ u = x,y]$ and the energy functional takes the form

$$\mathcal{F} = -\frac{\hbar j_c\phi_0}{2\pi edH}\sum_{m,n}\Big\{\sin\Big[\frac{\pi dH}{\phi_0}(X_{m+1,n} - X_{mn})\Big]$$

$$\times\cos\Big[\varphi_{m,n-1} - \varphi_{mn} + \frac{\pi dH}{\phi_0}(X_{m+1,n} + X_{mn}) + \alpha_x\Big]$$

$$+\sin\Big[\frac{\pi dH}{\phi_0}(Y_{m,n+1} - Y_{mn})\Big]\cos\Big[\varphi_{mn} - \varphi_{m-1,n} + \frac{\pi dH}{\phi_0}(Y_{m,n+1} + Y_{mn}) + \alpha_y\Big]\Big\},$$

where X_{mn} and Y_{mn} are the projections of the radius vector of a site of the fundamental lattice, and the index on the phase φ refers to the dual lattice (Fig. 1). The phases α_x and α_y are fixed by the external current. A distinction between (4) and the functional of the XY model is the oscillatory H dependence of the coupling force (of the total Josephson current) between the granules $[\sim H^{-1}\sin(\gamma H)]$. A behavior of this sort can lead to a strong H dependence of T_c in model (4) (Ref. 8). Some particularly nontrivial consequences for (4) can be found for random lattices, in which the sign of the coupling force can vary from junction to junction.

In conclusion we wish to emphasize that the system described above incorporates the effect of the self-field of the Josephson currents [see functional (2)], while the field and the coupling force between granules are usually assumed to be given in the XY model, and the problem reduces to one of determining φ. The approach outlined above can be generalized without any difficulty to lattices of other types and also to disordered systems.

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[1]Proceedings of the NATO Advanced Research Workshop on Coherence in Superconducting Networks, Delft, 1987, North-Holland, Amsterdam, 1988, pp. 1-302; Physica **B152**, 1-302 (1988).
[2]L. B. Ioffe and A. I. Larkin, Zh. Eksp. Teor. Fiz **81**, 707 (1981) [Sov. Phys. JETP **54**, 378 (1981)].

[3]C. J. Lobb, D. W. Abraham, and M. Tinkham, Phys. Rev. B **27**, 150 (1983).
[4]É. B. Sonin, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 415 (1988) [JETP Lett. **47**, 496 (1988)].
[5]P. Bak and T. Bohr, Rep. Prog. Phys. **45**, 587 (1982).
[6]A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect*, Wiley-Interscience, 1981.
[7]J. F. Kwak, E. L. Venturini, P. J. Nigrey, and D. S. Ginley, Phys. Rev. B **37**, 9749 (1988).
[8]C. Giovahhella, L. Fruchter, and C. Chappert, Ann. Phys. (Paris) **13**, 393 (1988).

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