Fluctuations in the intensity of 1/f noise in disordered metals

B. L. Al'tshuler and B. Z. Spivak
B. P. Konstantinov Institute of Nuclear Physics, Academy of Sciences of the USSR

(Submitted 20 March 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **49**, No. 8, 459–462 (25 April 1989)

It is shown that the spectrum of a nonequilibrium low-frequency electric noise in a macroscopic sample of a disordered metal fluctuates appreciably from one sequence to another when the measuring contacts are small. The classical and quantum-mechanical contributions to these fluctuations are considered.

1. It is widely believed that $1/f$ noise occurs as a result of the presence in a sample of so-called two-level systems[1]—atoms or groups of atoms which can be in two relatively stable states. Since these states of the two-level systems scatter electrons in different ways, the transitions between them change the resistance of the sample with time, i.e., they produce a nonequilibrium electric noise. It is natural to assume that these transitions occur randomly, with a time scale, τ_j for each two-level system. The cross section for the scattering of jth two-level system by an electron, $s_j(t) = S_{j0} + \delta s_j(t)$, will then be subjected to a temporal fluctuation with a Lorentzian spectrum:

$$\langle \delta s_j(t_1)\delta s_k(t_2)\rangle_\omega \equiv \lim_{\theta\to\infty} (8\pi i\theta)^{-1}\int_{-\theta}^{+\theta} d(t_1+t_2)\int_{-\infty}^{\infty} d(t_1-t_2)\delta s_j(t_1)\delta s_j(t_2)$$
$$\times\ \exp[i\omega(t_1-t_2)] \propto \delta_{jk}\,\tau_j\,[(\omega\tau_j)^2+1]^{-1}. \quad (1)$$

A simple mechanism (let us call it a classical mechanism) by which the resistance changes involves a change in the mean free path near the jth two-level system, i.e., a fluctuation of the local conductivity. On the other hand, the residual resistance of the sample is highly sensitive to the change in the random potential because of the quantum interference.[2,3]

If the sample has just one two-level system, the time evolution of the resistance, $R(t) = R_0 + \delta R(t)$, is of the form of a "telegraph signal": $R(t)$ has only two values, and the fluctuations have a Lorentzian spectrum. In a small sample, $R(t)$ is a collection of several telegraph signals, and $\langle \delta R(t_1)\delta R(t_2)\rangle_\omega$ is a sum of the Lorentzian lines.[4,5] If the sample has many two-level systems with $R(t)$ components of the same order, the noise spectrum is determined by a $P(\tau_j)$ profile of the relaxation times τ_j

$$\langle \delta R(t_1)\delta R(t_2)\rangle_\omega \propto \int P(\tau)[(\omega\tau)^2+1]^{-1}\,\tau d\tau. \quad (2)$$

In a very broad interval $\tau_{\min} < \tau < \tau_{\max}$ the $P(\tau)$ profile is assumed to have the form

$$P(\tau) = g\tau^{-1}; \qquad g = [\ln(\tau_{max}/\tau_{min})]^{-1} \quad (3)$$

 0021-3640/89/080527-05$01.00

After substituting (3) in (2), this relation becomes the $1/\omega$ law for the noise spectrum.

However, even if the size of the sample tends toward infinity, the $1/\omega$ law does not always hold. We will show that when the standard four-contact method is used to measure the resistance, the fluctuation spectrum $\delta R(t)$ is a random spectrum which depends essentially on the position of the two-level system if the contacts are small.

2. We will begin with a classical problem. The local conductivity

$$\sigma(\mathbf{r},t)=\sigma_0+\sum_j \delta\sigma_j(t)\,\delta(\mathbf{r}-\mathbf{r}_j), \tag{4}$$

corresponds to a set of two-level systems with relaxation times τ_j at the points $\mathbf{r}_j$, where $\delta\sigma_j(t)$ are random time-dependent functions with a Lorentzian spectrum

$$\langle \delta\sigma_j(t_1)\delta\sigma_j(t_2)\rangle_\omega=\delta_{jk}\beta\tau_j\,[(\omega\tau_j)^2+1]^{-1}\ . \tag{5}$$

The local electric field $\mathbf{E}(\mathbf{r})=\mathbf{E}_0+\delta\mathbf{E}(\mathbf{r})$ and the current $\mathbf{j}(\mathbf{r})=\mathbf{j}_0+\delta\mathbf{j}(\mathbf{r})$ are determined from a continuity equation $\operatorname{div}\mathbf{j}=0$ which, after directing the applied electric field $\mathbf{E}_0$ along the x axis, can be written as

$$\operatorname{div}\delta\mathbf{j}(\mathbf{r})=E_0\frac{\partial}{\partial x}\sigma(\mathbf{r},t)+\sigma_0\operatorname{div}\delta\mathbf{E}(\mathbf{r})=0. \tag{6}$$

It follows from (4)–(6) that fluctuations of the electric potential $\delta\varphi(\mathbf{r},t)$ and the spectrum of their noise are expressed in terms of the Green's function of the Laplace equation $G(\mathbf{r},\mathbf{r}')$

$$\delta\varphi(\mathbf{r},t)=E_0\sigma_0^{-1}\sum_j\delta\sigma_j(t)\frac{\partial}{\partial x}G(\mathbf{r},\mathbf{r}_j)$$

$$K_\omega(\mathbf{r},\mathbf{r}')\equiv\langle\delta\varphi(\mathbf{r},t_1)\delta\varphi(\mathbf{r}',t_2)\rangle_\omega=\left(\frac{E_0}{\sigma_0}\right)^2\sum_j\beta\tau_j\frac{1}{(\omega\tau_j)^2+1}\frac{\partial}{\partial x}G(\mathbf{r},\mathbf{r}_j)\frac{\partial}{\partial x'}G(\mathbf{r}',\mathbf{r}_j). \tag{7}$$

This spectrum depends on the arrangement of the two-level system $\{\mathbf{r}_j\}$, i.e., it is the random function ω. Averaging over $\{\mathbf{r}_j\}$, we find the average spectrum $\overline{K}_\omega$ and its autocorrelation function $Z_{\omega\omega'}(\mathbf{r},\mathbf{r}')\equiv\overline{K_\omega(\mathbf{r},\mathbf{r}')K_{\omega'}(\mathbf{r},\mathbf{r}')}-\overline{K}_\omega(\mathbf{r},\mathbf{r}')\overline{K}_{\omega'}(\mathbf{r},\mathbf{r}')$ (the overbar means that the averaging is over $\{\mathbf{r}_j\}$). If L is the size of the sample, and n is the concentration of the two-level systems which undergo transitions, we have

$$\overline{K}_\omega=g\beta nE_0^2\sigma_0^{-2}I_1(\mathbf{r},\mathbf{r}')\frac{1}{\omega}\ ;$$

$$Z_{\omega,\omega'}=g\beta^2nE_0^4\sigma_0^{-4}\left\{\ln\left(\frac{\omega}{\omega'}\right)\frac{I_2}{\omega^2-\omega'^2}-\frac{gI_1^2}{\omega\omega'}L^{-d}\right\} \tag{8}$$

$$I_n(\mathbf{r},\mathbf{r}')\equiv\int[\partial G(\mathbf{r},\mathbf{r}_0)/\partial x]^n\,[\partial G(\mathbf{r}',\mathbf{r}_0)/\partial x]^n d\mathbf{r}_0\,.$$

In the three-dimensional case ($d=3$) $G(\mathbf{r},\mathbf{r}') = (4\pi|\mathbf{r}-\mathbf{r}'|)^{-1}$ and the integrals over $\mathbf{r}_0$ in (8) diverge in the lower limit. At $\mathbf{r}=\mathbf{r}'$ these integrals should be cut off at the length scale of the contact, a. If $L\to\infty$, the second term in the braces in (8) can be dropped. As a result, we have

$$\overline{K}_\omega(\mathbf{r}=\mathbf{r}') \propto E_0^2\,\beta n\,(4\pi\sigma_0)^{-2}\,\frac{1}{\omega a}\,[\ln(\frac{\tau_{max}}{\tau_{min}})]^{-1} \tag{9}$$

$$Z_{\omega,\omega'}(\mathbf{r}=\mathbf{r}') \propto (\frac{E_0}{4\pi\sigma_0})^4\,\frac{\beta^2 n}{a^5}\,[\ln(\frac{\tau_{max}}{\tau_{min}})]^{-1}\,\frac{1}{\omega^2-\omega'^2}\ln(\frac{\omega}{\omega'})\,. \tag{10}$$

According to (9) and (10), the fluctuations of the spectrum are much stronger than the average $Z_{\omega\omega'} \gg \overline{K}_\omega^2$ if the condition $na^3 \ll \ln(\tau_{\max}/\tau_{\min})$ is satisfied.

The fluctuations are strong because of the rapid decay of $G(\mathbf{r}-\mathbf{r}_j)$. The noise spectrum at the frequency ω is determined by the two-level system with $\tau \approx \omega^{-1}$ which is closest to the observation point. Within the range of radius $ng^{-1/3}$ there is roughly one two-level system [see Eq. (3)] with a relaxation time in the interval τ, 2τ. This two-level system determines K_ω at $\omega\tau \approx 1$. The randomness in the arrangement of these two-level systems gives rise to random oscillations of the noise spectrum K_ω with a characteristic "period" $\Delta\omega \approx \omega$ (Fig. 1). As the size of the contact a increases, the oscillation amplitude decreases and at $a > ng^{-1/3}$, when K_ω is affected simultaneously by many two-level systems, it drops below $\overline{K}_\omega$.

In the two-dimensional case ($d=2$) $G(\mathbf{r}-\mathbf{r}_j) = (2\pi)^{-1}\ln|\mathbf{r}-\mathbf{r}_j|$ and the integral I_1 in (9) is logarithmic. If n is now a two-dimensional density of the two-level system, we have

$$\overline{K}_\omega(\mathbf{r}=\mathbf{r}') \propto (\frac{E_0}{2\pi\sigma_0})^2\,\beta n\,[\ln(\frac{\tau_{max}}{\tau_{min}})]^{-1}\,\frac{1}{\omega}\ln(\frac{L}{a})$$

$$Z_{\omega,\omega'}(\mathbf{r}=\mathbf{r}') \propto (\frac{E_0}{2\pi\sigma_0})^4\,\beta^2 n\,[\ln(\frac{\tau_{max}}{\tau_{min}})]^{-1}\,\frac{1}{a^2(\omega^2-\omega'^2)}\ln(\frac{\omega}{\omega'})\,. \tag{11}$$

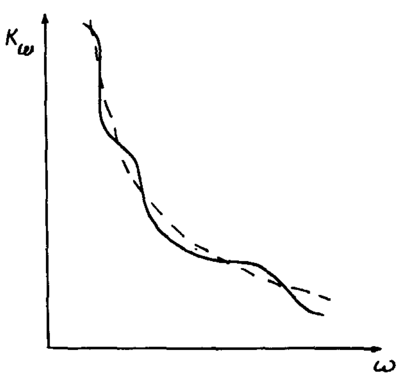

FIG. 1. Noise spectrum for a specific K_ω (solid curve) and one that is averaged over $\overline{K}_\omega \propto \omega^{-1}$ (dashed curve).

We see that at $d=2$ the fluctuations are significantly weaker than those at $d=3$: The condition for strong fluctuations, $nga^2 \ln(L/a) \ll 1$, is always broken in the limit $L \to \infty$, although K_ω may differ dramatically from $\overline{K}_\omega$ even when L is very large. The point here is that at $d=2$ the total contribution from the two-level systems, which are situated a distance ranging from r to $2r$ from the contact, to K_ω does not depend on the value of r.

With regard to the correlation of $\delta\varphi$ at different points r and r', we see that $Z_{\omega,\omega'}(\mathbf{r},\mathbf{r}')$ with $|\mathbf{r}-\mathbf{r}'| \gg a$ differs from (1) and (11) only in that a is replaced by $|\mathbf{r}-\mathbf{r}'|$, i.e., $Z \propto |\mathbf{r}-\mathbf{r}'|^{4-3d}$.

3. Quantum-mechanical effects cannot be described in terms of the local conductivity, but at nonzero temperature T and with $L \to \infty$ the sample can be partitioned into cubes which behave as individual classical resistors. The minimum size L_0 of such a cube is either the length $L_T = (hD/T)^{1/2}$ (where $D = l\tau_e/d$ is the diffusion coefficient, and l and τ_e are the mean free path and the mean free time of an electron) or the length scale for the loss of phase coherence, L_φ. The conductance of the cubes of length L_0 is spread over an interval on the order of e^2/h (Refs. 6 and 7).

The "fast" two-level systems, i.e., those for which $\omega\tau \ll 1$, also contribute to the loss of phase coherence. In this case L_0 is defined by the condition that the number of fast two-level systems in a volume L_0^d be equal to the number of impurities, $N_0 = lL_0^{d-2}/\delta s$, whose change in the cross section by δs changes the conductance of the cube by e^2/h (Refs. 2 and 3). Since there are $\ln(1/\omega\tau_{\min}) \gg 1$ times fewer two-level systems with the relaxation times in the interval ω^{-1}, $2\omega^{-1}$ than there are fast two-level systems, i.e., in a volume L_0^d they are heavily outnumbered by N_0, these two-level systems contribute separately to $\delta\varphi$: The probability that an electron will be scattered by two two-level systems of this sort, as it passes through the cube, is negligible. At $a \gg L_0$, Eqs. (7)–(11) are therefore valid and the quantum effect in β is on the order

$$\beta_q \approx e^2 L_0^4 / h^2 N_0 \qquad (a \gg L_0), \tag{12}$$

since the conductance of the cube in a single two-level system changes by e^2/h as a result of the transition.

From the viewpoint of the classical effects, the size of the elementary cube is l, and the relative change in its conductance as a result of the transition in one two-domain system is $\approx \delta s l^{1-d}$. Consequently, the classical contribution to β is on the order

$$\beta_c \approx \left(\frac{e^2}{h} \frac{\delta s}{\lambda^{d-1}} l^2\right)^2 \tag{13}$$

where λ is the electron wavelength. The quantum-mechanical contribution to β, in contrast with the classical contribution, is linear in δs. According to (12) and (13), the quantum-mechanical effects with $(L_0/l)^{6-d} > l^{d-1}\delta s \lambda^{2-2d}$ are responsible for the noise (see, e.g., Ref. 8).

If $a < L_0$, the quantum-mechanical effect on the noise increases in the three-dimensional case in comparison with that in (12) by a factor of L_0/a. This increase is attributable to the fact that at the length scales $< L_0$ a change in the conductivity due to a single two-level system is not a δ-function of the type in (4) but is a power-law decay.[9]

[1]Sh. M. Kogan, Usp. Fiz. Nauk **145**, 285 (1985) [Sov. Fiz. Usp. **28**, 170 (1985)].
[2]B. L. Al'tschuler and B. Z. Spivak, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 363 (1985) [JETP Lett. **42**, 447 (1985)].
[3]S. Feng, P. A. Lee, and A. D. Stone, Phys. Rev. Lett. **56**, 1960 (1986).
[4]R. S. Ralls and R. A. Buhman, Phys. Rev. Lett. **60**, 2434 (1988).
[5]K. E. Track and D. E. Prober, Jpn. J. Appl. Phys. **26**, S3, 1635 (1987).
[6]P. A. Lee and A. D. Stone, Phys. Rev. Lett. **55**, 1622 (1985).
[7]B. L. Al'tschuler, Pis'ma Zh. Eksp. Teor. Fiz. **41**, 530 (1985) [JETP Lett. **41**, 648 (1985)].
[8]Yu. M. Gal'perin, V. G. Karpov, and V. I. Kozub, Zh. Eksp. Teor. Fiz. **95**, 1123 (1989) [Sov. Phys. JETP **68**, 648 (1989)].
[9]A. Yu. Zyuzin and B. Z. Spivak, Zh. Eksp. Teor. Fiz. **93**, 994 (1987)[Sov. Phys. JETP **66**, 560 (1987)].

Translated by S. J. Amoretty