Quantum effects in the formation of population inversions and lasing in germanium hot-hole system

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Quantum effects stemming from a mixing of the states of light and heavy holes in *p*-Ge in fields **E**⊥**H** are shown to play an important role in the formation of population inversions and lasing in the cases of both intraband and interband hole transitions. These effects become sharply more important when the anisotropy of the valence band is taken into account.

A semiclassical treatment of the motion of the charge carriers has been used in most of the previous theoretical work on the formation of population inversions in nonequilibrium semiconductor plasmas in strong electric and magnetic fields.[1] Recent experimental and theoretical results, however, demonstrate the need to incorporate phenomena which stem from the quantizing effects of fields.[2–6]

In the present letter we show, on the basis of some new theoretical and experimental work, that quantum-mechanical effects in semiconductors with degenerate bands, such as *p*-Ge, play an important role in these processes and not only give rise to discrete lasing branches but also cause a radical change in the entire picture of phenomena involved in the formation of the population inversions and the lasing in the hot-carrier system in fields **E**⊥**H**. In the approximation of spherical bands, the role played by these effects is a minor one, but it becomes sharply more important when the complex anisotropic structure of the Ge valence band is taken into account.

Our analysis is based on the eigenfunctions and energy spectrum of holes in germanium in fields **E**⊥**H** which we have calculated by the methods of Refs. 3 and 6. We used the complete Luttinger Hamiltonian, which incorporates the anisotropy of the Ge valence band. As in the isotropic spherical model,[3,6] we observed a repulsion of

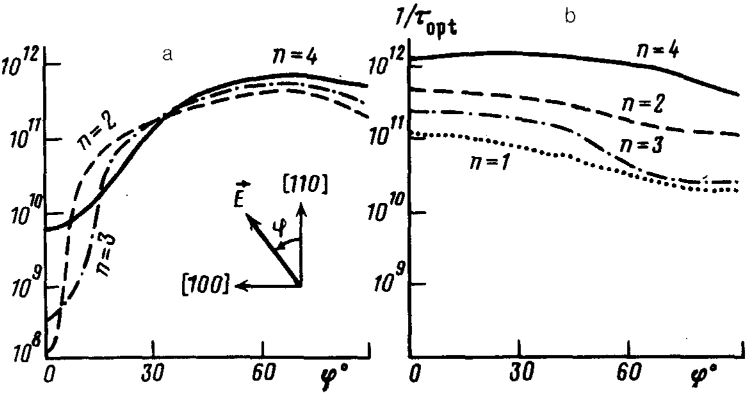

FIG. 1. Theoretical behavior of $1/\tau_{\rm opt}$ as a function of the angle φ for various Landau levels of light holes of the $n = 1, 2, 3, 4$ series. a—$M_J = +3/2, -1/2$; b—$M_J = -3/2, +1/2$ ($H = 24$ kOe, $E = 2.4$ kV/cm, $\varepsilon^0_{dr}/\hbar\omega^0_c = 102$, $p_z = 0$).

the Landau levels of light and heavy holes due to an interaction of hole states with identical total energy. The overall picture of the levels and the nature of their interaction vary significantly with the crystallographic orientation of the fields.

Knowing the energy spectrum and wave functions, we can calculate the lifetime of the light holes in mixed states, whch is set by their scattering by optical phonons, $\tau_{\rm opt}$. The probability for the scattering of a hole in an initial state n, p_z, p_x was calculated by summing the probabilities for the scattering to all final states n', p'_z, p'_x accompanied by the emission of an optical phonon:

$$W_{opt} \equiv {}^1/_{\tau_{opt}} = \frac{2\pi}{\hbar} \sum_{\mathbf{u}} \sum_{\mathbf{q}} \sum_{n', p'_z, p'_x} |\langle n', p'_z, p'_x | \hat{H}_{opt} \exp(-i\mathbf{q}\mathbf{r}) | n, p_z, p_x \rangle|^2$$

$$\times\ \delta(E_{n', p'_z, p'_x} - E_{n, p_z, p_x} + \hbar\omega_0).$$

Here $\hat{H}_{\rm opt}$ is a 4×4 matrix strain-energy operator[7]; n, p_z, p_x specify the Landau level and the components of the generalized quasimomentum of the hole (**H**||**Z**,**E**||**Y**); and $\hbar\omega_0$, **q**, and **u** are the energy, quasimomentum, and polarization vector of the optical phonon. Data on $1/\tau_{\rm opt}$ are shown in Fig. 1 for the configuration **H**||[110], for various directions of the vector **E** in the (110) plane. In the interval $\varphi = 0$–90°, where φ is the angle between **E** and the $[1\bar{1}0]$ axis, this configuration spans essentially the entire set of values of the effective mass of heavy holes in the drift direction, which is perpendicular to **E** and **H**.

Figure 2 shows experimental results on the observation of lasing of two types in Ge: on transitions between subbands of light and heavy holes and on intraband cyclotron transitions of light holes. This study was carried out in the configuration adopted in the calculations. We measured the regions of the fields **E** and **H**, in which lasing

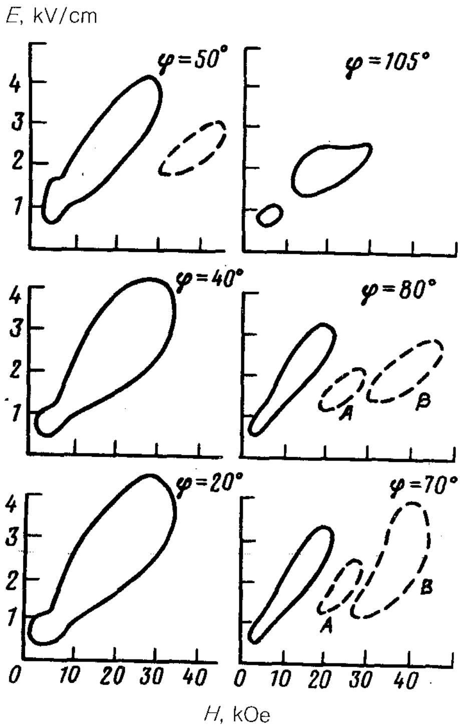

FIG. 2. Experimental regions in the E, H plane in which lasing occurs on intersubband transitions (solid lines) and on intraband cyclotron transitions (dashed lines) in Ge at various angles φ.

occurred, and the spectrum of the emission. The experimental procedure was similar to that described in Refs. 3 and 4. Rectangular samples of identical dimensions were cut from a single Ge bar ($N_A = 7\times10^{13}$ cm^{-3}). The applied electric field made various angles with the $[1\bar{1}0]$ axis. The values of φ were found experimentally, with allowance for the Hall component. It can be seen from Fig. 2 that the conditions required for the occurrence of lasing on the intersubband and cyclotron transitions are different. For lasing on the intersubband transitions, the orientations of **E** which are the optium orientations from the standpoint of the emission power and the width of the regions of the fields **E** and **H** are close to $\varphi\approx0°$ ($\mathbf{E}\|[1\bar{1}0]$); for the cyclotron lasing, the optimum orientations are close to $\varphi\approx70°$.

These results do not correspond to the predictions of the semiclassical model, while they can be explained satisfactorily in a model which incorporates quantum-mechanical effects. Specifically, from the semiclassical standpoint the optimum orientation for lasing on intersubband transitions should be that corresponding to the maximum difference between the masses of the light and heavy holes in the drift direction ($\varphi\approx55°$, $\mathbf{v}_{\rm dr}\|[111]$) (Ref. 2). In the experiments, the optimum orientation of **E** was different ($\varphi\approx0°$) and corresponded to the predictions of the quantum-mechanical calculations. It can be seen from Fig. 1 that the calculated value of $1/\tau_{\rm opt}$ depends only weakly on φ for the series $M_J = -3/2, +1/2$, but for the series $M = 3/2, -1/2$

there is a significant change; the value decreases rapidly toward $\varphi \approx 0°$. This result implies an increase in the lifetime of the light holes; such an increase would promote the formation of a population inversion and the onset of lasing on intersubband hole transitions. The physics underlying these features stems from the quantum-mechanical effect of the interaction of the states of light and heavy holes in fields $\mathbf{E} \perp \mathbf{H}$. As a result of the interaction with the heavy subband and the hybridization of the wave functions, the "mass of the light holes"[3] increases in the mixed states, giving rise to an increase in the scattering of these holes by optical phonons and an increase in $1/\tau_{\rm opt}$. The effect of the interaction is at a minimum at angles $\varphi \approx 0°$. This orientation turns out to be the optimum orientation for intersubband hole transitions. A governing role is also played by the absolute value of the effective mass of the admixed heavy holes in the drift direction; in the orientation $\varphi \approx 0°$ this effective mass is significantly smaller ($m_{\rm dr}^{\rm hh} \approx 0.21 m_0$) than in other orientations ($m_{\rm dr}^{\rm hh} = 0.3$–$0.49$ at $\varphi = 40$–$90°$).

The same quantum-mechanical effects, due to the interaction of light and heavy holes, turn out to be important in the redistribution of carriers in the "capture" region between Landau levels in the light-hole subband. Here their nature is similar to that discussed in Refs. 8 and 9. In this regard, the series $+3/2$, $-1/2$ is of no particular interest (Fig. 1). On the other hand, the results calculated for the series $-3/2$, $+1/2$ reveal some fundamental distinctive features. The frequency of the scattering of light holes by the $n = 3$ level turns out to be significantly lower than that for scattering by

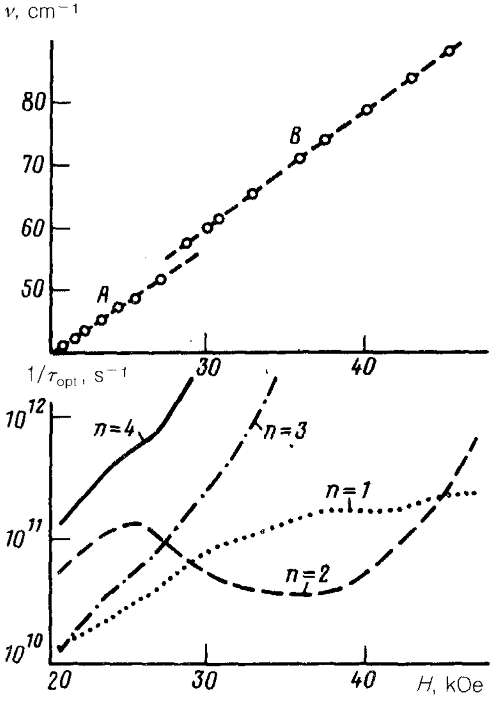

FIG. 3. Theoretical behavior of $1/\tau_{\rm opt}$ as a function of the magnetic field [for the series $M_J = -3/2, +1/2$, $\varphi = 90°$, $E/H = 0.1$ kV/(cm·kOe)] and experimental field dependence of the frequency, $\nu(H)$, of the lasing on cyclotron transitions in lasing regions A and B (Fig. 2).

the $n = 2$ level, and the difference increases with the deviation from $\varphi = 0°$. The inversion which arises in the populations of these levels apparently determines the nature of the cyclotron lasing on the $3 \rightarrow 2$ transitions. The optimum orientations from the standpoint of the onset of this lasing should be orientations with $\varphi \geqslant 50°$, just as we see experimentally. The data on Fig. 1 correspond to fields $H = 24$ kOe. It can be seen from Fig. 3 that the general picture of the scattering and the populations of the Landau levels changes with increasing magnetic field. An inversion of the carrier populations in levels 3 and 2 becomes impossible, while an inversion for levels 2 and 1, in contrast, becomes possible. This switching of the lasing is seen experimentally. Each type of lasing, $A(3 \rightarrow 2)$ and $B(2 \rightarrow 1)$, occurs in a corresponding region in the E, H plane (Figs. 2 and 3), in agreement with the theoretical results. The different types of lasing differ in the field dependence of the frequency, $\nu(H)$, since the Landau levels in semiconductors with degenerate bands are not equidistant.

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