Excess quantum noise in $2D$ ballistic point contacts

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It is shown that if the matrix of the transmission coefficients t_{nm}, which describes the transport, is diagonal and if the t_{nn} are equal to unity or zero, and the conductance G is quantized exactly in units of $e^2/\pi\hbar$, the excess noise is absent if the inelastic processes are ignored. The spectral intensity of the excess noise $s_v(\omega)$ is therefore at a maximum in a narrow range of parameters which correspond to the transition from $G = (e^2/\pi\hbar)n$ to $G = (e^2/\pi\hbar)(n+1)$.

Let us consider a point contact which consists of a microscopic constriction in a $2D$ electron layer with a smooth interface separating a conducting region and an insulating region (Fig. 1). Such point contacts are used in experimental studies of quantization of the resistance as a function of the width d(Refs. 1 and 2). It was shown in Ref. 3 that in such a geometry the variables in the Schrödinger equation can be separated in the adiabatic approximation and the conductance G can be expressed in the multichannel Landauer formula[4] in terms of the sum of the transition coefficients in each channel (when $k_b T = 0$) in the following way: $G = (e^2/\pi\hbar)\Sigma_n D_n(\epsilon_F)$. The coefficients $D_n(\epsilon_F)$ were calculated as functions of the width d and the radius of curvature of the microconstriction, R. The plots which were obtained explain the experimentally observed steplike plot of G vs the width d. On the plateau the conductance could be described by $G = (e^2/\pi\hbar)n$, where n is an integer. This relation corresponds to the case in which there are n completely open channels and the remaining channels are closed.

In the present letter we will show that the spectral density of excess current noise can be expressed in terms of the same transition coefficients $D_n(\epsilon)$ and that there is no noise in the special case where all the D_n are either 0 or 1 if the inelastic processes are ignored. (Excess noise is said to be that noise which is produced when a finite current

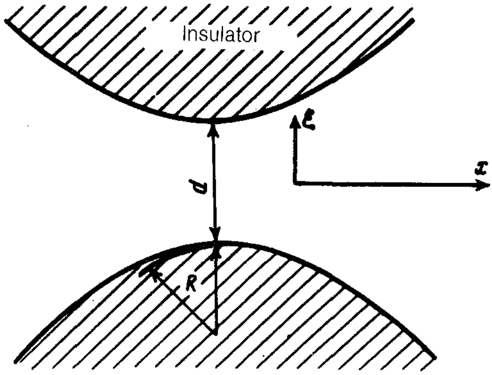

FIG. 1.

flows in addition to the equilibrium Nyquist current.)

The total-current operator

$$\hat{I}(x,t) = \frac{ie\hbar}{2m}\int d\xi(\vec{\nabla}\hat{\Psi}^{+}(\mathbf{r},t)\hat{\Psi}(\mathbf{r},t) - \hat{\Psi}^{+}(\mathbf{r},t)\vec{\nabla}\hat{\Psi}(\mathbf{r},t)) \tag{1}$$

can be expressed in terms of $\hat{\Psi}$ operators of the system which are given by

$$\hat{\Psi} = \sum_n \int \frac{dp}{\pi\hbar}\hat{a}_n(p)e^{\frac{ipx}{\hbar} - \frac{i}{\hbar}(\frac{p^2}{2m} - \mu - \frac{eV}{2})t}\varphi_n(\xi)A_n(\frac{p^2}{2m} + \frac{eV}{2})$$

$$+ \sum_n \int \frac{dp}{\pi\hbar}\hat{b}_n(p)e^{-\frac{i}{\hbar}(\frac{p^2}{2m} - \mu + \frac{eV}{2})t}\varphi_n(\xi)\{e^{-i\frac{p}{\hbar}x} + B_n(\frac{p^2}{2m} - \frac{eV}{2})e^{i\frac{p}{\hbar}x}\}$$

$$\hat{\Psi}^{+} = \sum_n \int \frac{dp}{\pi\hbar}\hat{a}_n^{+}(p)e^{-i\frac{p}{\hbar}x + \frac{i}{\hbar}(\frac{p^2}{2m} - \mu - \frac{eV}{2})t}\varphi_n^{*}(\xi)A_n^{*}(\frac{p^2}{2m} + \frac{eV}{2}) \tag{2}$$

$$+ \sum_n \int \frac{dp}{\pi\hbar}\hat{b}_n^{+}(p)e^{\frac{i}{\hbar}(\frac{p^2}{2m} - \mu + \frac{eV}{2})t}\varphi_n^{*}(\xi)\{e^{i\frac{p}{\hbar}x} + B_n^{*}(\frac{p^2}{2m} - \frac{eV}{2})e^{-i\frac{p}{\hbar}x}\}.$$

Here μ is the chemical potential of the electrons, V is the potential difference at the edges, A_n and B_n are the amplitudes of the transmitted and reflected waves in the corresponding channel, $\hat{a}^{+}$ and $\hat{a}$ are the operators for the creation and annihilation of particles in the left reservoir, and $\hat{b}^{+}$ and $\hat{b}$ are the operators for the creation and annihilation of particles in the right reservoir.

The expression for the spectral density of excess noise

$$S_v(\omega) = \int dt e^{i\omega t}(\frac{1}{2}\langle\hat{I}(t)\hat{I}(0) + \hat{I}(0)\hat{I}(t)\rangle - \langle\hat{I}\rangle^2) - S_0(\omega)$$

[the angle brackets denote averaging over the density matrix, and $s_0(\omega)$ is the spectral density of the equilibrium sound] which we are seeking has the form

$$S_v(\omega) = \frac{e^2}{\pi^2\hbar^2} \sum_{+,-,k} \int d\epsilon \, \{n^a(\epsilon \pm \omega)(1 - n^a(\epsilon))D_k(\epsilon \pm \omega)D_k(\epsilon)$$

$$+ n^a(\epsilon \pm \omega)(1 - n^b(\epsilon))D_k(\epsilon \pm \omega)(1 - D_k(\epsilon)) + n^b(\epsilon \pm \omega)(1 - n^a(\epsilon))D_k(\epsilon)(1 - D_k(\epsilon \pm \omega))$$

$$+ n^b(\epsilon \pm \omega)(1 - n^b(\epsilon))D_k(\epsilon \pm \omega)D_k(\epsilon)\} - S_0(\omega). \quad (3)$$

Here $n^a(\epsilon) = [\exp[(\epsilon - eV/2)/k_B T] + 1]^{-1}$, $n^b(\epsilon) = [\exp[(\epsilon + eV/2)/k_B T] + 1]^{-1}$, and $D_k(\epsilon) = |A_k(\epsilon)|^2$. If the temperature T is equal to zero, and $eV \ll \Delta E$, where ΔE is the length scale along which the transmission level changes, we find from (3) the following expression for the lowest frequencies:

$$S_v(\omega = 0) = \frac{e^2}{\pi^2\hbar} eV \sum_n D_n(1 - D_n). \quad (4)$$

If all $D_n \ll 1$, this expression describes the ordinary shot noise: $S_v(\omega = 0) = e\langle I \rangle/\pi$. If, on the other hand, all of the coefficients D_n are either 0 or 1, then there is no excess noise. The $D_n(\epsilon)$ curves obtained in Ref. 3 have the form $D_n(\epsilon) = (1 + \exp(-\epsilon/\Delta E_n))^{-1}$, where $\Delta E_n = \hbar^2 n/\sqrt{2Rd^3}m$. We see that the nonvanishing component of S_v appears only when $\mu \sim E_n$. If $\mu = E_n$, the following expressions for various limiting cases can be obtained:

$$S_v(\omega = 0) = \frac{e^2}{\pi^2\hbar} \frac{(eV)^2}{4k_B T} \qquad (eV \ll k_B T \ll \Delta E_n) \quad (5)$$

$$S_v(\omega = 0) = \frac{e^2}{\pi^2\hbar} \frac{(eV)^2}{6\Delta E_n} \qquad (eV, \Delta E_n \ll k_B T). \quad (6)$$

The frequency decay scale of $S_v(\omega)$ is the smallest quantity in $eV/\hbar$, $k_B T/\hbar$, and $\Delta E_n/\hbar$.

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