"Blocking" of a plasma wave by a relativistic electron bunch and its use in collective acceleration

V. A. Bazylev and V. V. Goloviznin
I. V. Kurchatov Institute of Atomic Energy, Moscow

(Submitted 10 April 1989)
Pis'ma Zh. Eksp. Teor. Fiz. **49**, No. 10, 545–548 (25 May 1989)

The excitation of wakefield oscillations by an extended bunch of relativistic electrons is analyzed. Self-consistent solitary solutions of the Bernstein-Greene–Kruskal type, for which the bunch is not slowed by the wakefield potential, are found. Substantial electric fields generated in the process are of interest for collective acceleration.

In connection with the problem of the collective acceleration of particles, Chen *et al.*[1,2] have suggested utilizing an electron bunch of small dimensions ($L \ll \omega_p^{-1}$ where ω_p is the plasma frequency, and here and below we are using $\hbar = c = 1$) to excite plasma waves. It was found, however, that although the amplitude (E^+) of the accelerating electric field in the wakefield wave increases with increasing number of particles in a bunch, $N(E^+ \sim Ne\omega_p^2)$, there is a simultaneous decrease in the lifetime of the bunch in proportion to N, since each particle of the bunch is acted on by a retarding field E^- of the same order of magnitude. Several subsequent studies[3,4] analyzed the excitation of a plasma wave by an extended bunch ($L \gtrsim \omega_p^{-1}$), with a given profile of the electron density along the longitudinal coordinate, in an effort to optimize the ratio $R = E^+/E^-$. In our opinion, that approach suffers from two serious shortcomings: First, the inverse effect of the plasma wave on the shape of the electron bunch is ignored. Second, the basic difficulty is still there: The bunch is still rapidly slowed as the result of an energy transfer to wakefield oscillations of the charge density.

In the present letter we show that there can be self-consistent, solitary, undamped solutions of the Bernstein-Greene–Kruskal (BGK) type in a beam-plasma system, which combine substantial accelerating fields, proportional to N, on the one hand, with the absence of a retardation of the electron bunch as a whole, on the other. This situation is achieved because the plasma wave is "blocked" by the electron bunch: It increases from the leading edge to the center and then decreases, vanishing at the trailing edge. There is no transfer of energy to the wakefield, so the ratio R effectively tends toward infinity.

We thus consider an extended electron bunch which is moving through a homogeneous plasma at a relativistic velocity $V_0 = 1 - \frac{1}{2}\gamma_0^{-2}$ ($\gamma_0 \gg 1$) along the z axis. As we stated above, we are interested in solutions which depend on the time only in the combination $\xi = z - V_0 t$. For simplicity, we ignore the transverse degrees of freedom at this point; i.e., we treat the beam as a train of infinitely thin, charged disks with a fixed charge distribution $\varphi(r_\perp)$ and transverse dimensions small in comparison with ω_p^{-1}. The function $\varphi(r_\perp)$ is normalized by the condition $\int d^2 r_\perp \varphi(r_\perp) = 1$. We also ignore the electrostatic repulsion, since its effect falls off in comparison with the interaction through the wakefield wave with increasing γ_0.

In this model, the force F which is exerted on a particle at the coordinate ξ is

$$F(\xi) = -Ne^2\omega_p^2\kappa \int_0^\infty d\Delta \cos(\omega_p\Delta) f(\xi + \Delta), \tag{1}$$

where $f(\xi)$ is the distribution of particles with respect to the longitudinal coordinate, normalized by

$$\int_{-\infty}^{+\infty} d\xi f(\xi) = 1; \tag{2}$$

here κ is a numerical factor, given by

$$\kappa = 2\int d^2r_\perp \varphi(\mathbf{r}_\perp) \int d^2r'_\perp \varphi(\mathbf{r}'_\perp) K_0(\omega_p|\mathbf{r}_\perp - \mathbf{r}'_\perp|). \tag{3}$$

To close this system of equations, we consider the motion of an individual particle of the bunch in collective field (1). The equations of motion

$$\begin{cases} \dfrac{d\xi}{dt} = \dfrac{1}{2}(\gamma_0^{-2} - \gamma^{-2}) & (4) \\[2ex] \dfrac{d(m\gamma)}{dt} = F(\xi) & (5) \end{cases}$$

can be solved in their general form. However, the solution takes its simplest form under the condition $|\gamma - \gamma_0| \ll \gamma_0$. Introducing $\nu = \gamma - \gamma_0$ and also the potential $v(\xi)$, in accordance with

$$F(\xi) = -\frac{m}{\gamma_0^3}\frac{dv}{d\xi}, \tag{6}$$

we find the integral of motion E:

$$E = \frac{\nu^2}{2} + v(\xi) = \text{const.} \tag{7}$$

The phase orbit of a particle has the simple form

$$\nu(\xi) = \pm\sqrt{2(E - v(\xi))}. \tag{8}$$

We are interested in localized solutions for which we have $f(\xi) \to 0$ and $F(\xi) \to 0$ as $\xi \to \pm\infty$. Restricting the analysis to the class of single-well potentials, and setting $v(\pm\infty) = 0$, we find the following system of equations to describe the undamped solution which we have been seeking:

$$
\left\{
\begin{aligned}
&\frac{dv(\xi)}{d\xi} = \alpha \int_0^\infty d\Delta \cos(\omega_p \Delta) f(\xi + \Delta) && (9)\\
&f(\xi) = \int_{v(\xi)}^{0} \frac{dE\, n(E)}{T(E)\sqrt{E - v(\xi)}} && (10)\\
&T(E) = \int_{\xi_1(E)}^{\xi_2(E)} \frac{d\chi}{\sqrt{E - v(\chi)}}, && (11)
\end{aligned}
\right.
$$

where $\alpha = Ne^2\omega_p^2\gamma_0^3\kappa/m$, and $\xi_1(E)$ and $\xi_2(E)$ are roots of the equation $v(\xi) = E$ $(\xi_1 < \xi_2)$.

Equations (10) and (11) are equations of the BGK type.[5] They determine $f(\xi)$ in terms of the new unknown function $n(E)$, which is the distribution of particles with respect to the variable E and which is normalized by the condition

$$
\int_{E_{min}}^{0} dEn(E) = 1. \qquad (12)
$$

The question in which we are interested thus reduces to the following: Do there exist normalizable functions $n(E)$ which provide the required asymptotic behavior of $f(\xi)$ and $F(\xi)$ and which furthermore satisfy the obvious physical requirements $f(\xi) \geqslant 0$ and $n(E) \geqslant 0$?

It is easy to verify that such solutions do exist, since system (9)–(11) can be solved explicitly. For any potential $v(\xi) \leqslant 0$, $v(\pm\infty) \to 0$, normalized by the condition

$$
\int_{-\infty}^{+\infty} d\xi v(\xi) = -\alpha\omega_p^{-2}, \qquad (13)
$$

one can specify functions

$$
f(\xi) = -\frac{1}{\alpha}\left(\frac{d^2 v}{d\xi^2} + \omega_p^2 v(\xi)\right) \qquad (14)
$$

and

$$
n(E) = \frac{T(E)}{2\pi}\left(\int_{-\infty}^{\xi_1(E)} \frac{d\xi}{\sqrt{v(\xi) - E}}\frac{df}{d\xi} - \int_{\xi_2(E)}^{\infty} \frac{d\xi}{\sqrt{v(\xi) - E}}\frac{df}{d\xi}\right), \qquad (15)
$$

which turn Eqs. (9)–(11) into identities and which have the required asymptotic behavior and normalization. The requirement that $f(\xi)$ and $n(E)$ be nonnegative imposes only a slight restriction on the class of permissible potentials $v(\xi)$.

In summary, we have described a new class of solitary, self-consistent solutions of the BGK type, which represent a packet of plasma waves which is moving at a relativistic velocity, along with the electron bunch which generates it. These solutions generate a longitudinal electric field of the same order of magnitude as that of a point charge, as can be verified easily on the basis of Eqs. (6) and (13). In contrast with a point charge, however, these solutions do not experience a retardation by the wake-field wave, so new opportunities are opened up for applications in collective acceleration of particles. The stability of these solutions is an extremely involved question and requires further research.

[1]P. Chen, R. W. Huff, and J. M. Dawson, Bull. Am. Phys. Soc. **29**, 1355 (1984).
[2]P. Chen, J. M. Dawson, R. W. Huff, and T. Katsouleas, Phys. Rev. Lett. **54**, 693 (1985).
[3]P. Chen and J. M. Dawson, in: Laser Acceleration of Particles, AIP Conference proceedings (eds. C. Joshy and T. Katsouleas), No. 130, 1985, p. 201.
[4]P. Chen *et al.*, Phys. Rev. Lett. **56**, 1252 (1985).
[5]I. B. Bernstein, G. M. Greene, and M. D. Kruskal, Phys. Rev. **108**, 546 (1957).

Translated by Dave Parsons