Plasma pinch as a source of cosmic rays

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The hypothesis that galactic cosmic rays are produced in cosmic plasma pinches leads to an accelerated-particle spectrum $dN/dE \sim E^{-\nu}$ with an index $\nu = 1 + \sqrt{3}$, which is approximately equal to the observed value $\nu_0 = 2.7$. The hypothesis therefore looks extremely plausible.

1. In the energy range 10^{10}–10^{20} eV, galactic cosmic rays have a spectrum $dN/dE \sim E^{-\nu}$ with an index of approximately $\nu_0 = 2.7$, but the existing theories for the production of galactic cosmic rays[1] do not predict such an index unambiguously.

In the present letter we solve the problem of the growth of nonlinear sausage perturbations in a relativistic cylindrical pinch with a complete skin effect. We derive a spectrum $\sim E^{-\nu}$ for the accelerated particles with an index $\nu = 1 + \sqrt{3}$, which is extremely close to the observed value and which is furthermore an unambiguous prediction. On the basis of this result it is hypothesized that the galactic cosmic rays are produced by precisely this mechanism. Plasma pinches have been under study in the laboratory for a long time now, and sausage perturbations are observed in them. A theory for these perturbations was derived in Refs. 2. Significantly, these experiments also reveal accelerated particles with energies on the order of 1 MeV, so the idea that similar processes may be occurring in space is a quite natural one. This idea was advanced by Gerlakh,[3] for example, but without any analysis. There is the question, however, of just where and how cylindrical plasma pinches, which should apparently be crossed by brief but extremely strong "cosmic lightning flashes," could form in space. Although this question requires further research, one might suggest as possible candidates some new astronomical entities which were recently observed: "jets," which are huge plasma streams moving at formidable velocities. These jets are observed for the most part in radio galaxies, e.g., NGC 6251, where a jet with a length on the order of 1 Mpc has been observed, but they are also found near stars. For example, the binary star SS433, which is a neutron star or a hole, provides a jet moving at a velocity of 80 000 km/s. At the "nodes" of a jet we observe a magnetic field directed perpendicular to the axis of the jet; the situation is extremely reminiscent of the necks in laboratory pinches. In our opinion, however, the most effective mechanism for the formation of cylindrical jets in space might be a process which starts with the collision of two plasma formations carrying oppositely directed "frozen-in" magnetic fields. A so-called neutral current sheet, in the form of a plane pinch, should arise at the interface. The tearing-mode instability would quickly tear this sheet up into current filaments: cylindrical pinches, in which necks could form and rupture the pinches, as is observed in laboratory pinches. This process could apparently occur repeatedly, since ruptured "z-pinches" can be seen fairly frequently in nebulae.

2. We consider an individual z-pinch, assuming for simplicity that it has a com-

plete skin effect, so we will describe the plasma in it by means of the well-known equations of single-fluid relativistic gasdynamics (Ref. 4, for example). These equations are difficult to solve in their three-dimensional form, so we approximate them by one-dimensional equations, using the familiar approximation of a narrow jet or channel, in which all quantities are assumed to remain constant over the cross section $S=\pi a^2$, where $a=a(t,z)$ is the pinch radius. In the proper frame of reference, moving at a velocity $v=v_z(t,z)$ along with the matter, we introduce the quantities h (the enthalpy density), n (the particle number density), and p (the pressure). We also introduce the unperturbed values $a_0 n_0$, and p_0. Assuming that the plasma is nonrelativistic in this proper frame, we use the standard relations $p=p_0(n/n_0)^s$, $s=C_P/C_V$, and $h=nM_0c^2$, where M_0 is the mass of an ion. We now introduce a dimensionless "density per unit length" $\rho_*=Sn/S_0n_0$ and assume that a constant current I_0 is flowing through the pinch. At the boundary $r=a(t,z)$ this current creates a field $B=2I_0/ca$ whose pressure $B^2/8\pi$ balances the plasma pressure, so we have $p=p_0(a_0/a)^2$. We can thus write the z component of the equation of motion and the continuity equation in the approximation of a narrow jet or channel in the following form, in our notation (see also Ref. 6):

$$\frac{\partial u}{\partial t}+\frac{c}{\gamma}u\frac{\partial u}{\partial z}=-\epsilon\Big(c\gamma\frac{\partial}{\partial z}+u\frac{\partial}{\partial t}\Big)\rho_*^{-1},\quad \frac{\partial}{\partial t}\gamma\rho_*+c\frac{\partial}{\partial z}u\rho_*=0, \tag{1}$$

where $\beta=v/c$, $u=\beta\gamma$, $\gamma=\sqrt{1-\beta^2}$, and $\varepsilon=sp_0/(s-1)n_0M_0c^2$ is a constant.

To solve relativistic system (1), we set $\rho_*=\varepsilon/x$, $u=\sinh y$ and then introduce the inverse functions $\varphi(x,y)$ and $\psi(x,y)$ in accordance with

$$ct=T(x,y)=(\psi\sinh y-\varphi\cosh y)xe^{-x},\quad z=Z(x,y)=(\psi\cosh y-\varphi\sinh y)xe^{-x}. \tag{2}$$

As a result, we find the two equations $\varphi'_y=\psi'_x+\psi/x$, $\psi'_y=x(\varphi-\varphi'_x)$. We thus have

$$\Delta^{(*)}\varphi=\varphi''_{yy}-2\varphi+\hat{L}\varphi=0,\quad \hat{L}\varphi=x\varphi''_{xx}+(2-x)\varphi'_x. \tag{3}$$

The eigenfunctions of the operator $\hat{L}$ are orthonormal Laguerre polynomials $\lambda_n=L_n^{(1)}(x)$ with a superscript 1. Since the set $\lambda_n(x)$ is complete, a general solution of (3) can be sought in the form of a series $\varphi=\Sigma_0^\infty\lambda_n f_n(y)$, where $f_n=C_n\exp(-|y|\sqrt{n+2})$. To determine the coefficients C_n, we note that the stage in which the necks develop in a pinch under natural conditions in space should be preceded by a stage of a comparatively quiet "plowing" of the plasma and the formation of the pinch itself. In our model we cannot deal with this preliminary stage, but we will simulate it by introducing the requirement that there be no perturbations in the time limit $t\to-\infty$; correspondingly, we are adopting the condition $\varphi\to\infty$ for $\rho_*=1$, $v=0$. In other words, the potential $\varphi(x,y)$ must have a singularity at the point $x=\varepsilon$, $y=0$, so it would be more legitimate to write Eq. (3) as if it were a Poisson equation $\Delta^{(*)}\varphi=-4\pi\rho_0$, where ρ_0 are δ-function "charges" which generate a potential and which lie at the point $x=\varepsilon$, $y=0$. The solution can then be written in terms of a Green's function:

$$\varphi = \int_0^\infty dx' \int_{-\infty}^{\infty} dy' \rho_0(x',y') G, \quad G = 2\pi w(x') S, \quad S = \sum_0^\infty \frac{\lambda_n(x)\lambda_n(x')}{\sqrt{n+2}} e^{-|y-y'|\sqrt{n+2}}. \qquad (4)$$

3. It should be kept in mind, however, that the charges ρ_0 lie only at the point $x=\varepsilon$, $y=0$, so it is convenient to switch to the new variables x_1, y_1, setting $x' = \varepsilon + x_1$, $y' = -y_1$. Using $w(x) = xe^{-x}$ and introducing $\rho_{\rm eff}(x_1,y_1) = 2\pi w(x')\rho_0$, we rewrite (4) in the more appropriate form

$$\varphi(x,y) = \int\int dx_1 dy_1 \rho_{eff} S(x; \varepsilon + x_1, y + y_1). \qquad (5)$$

We expand the function S in a Taylor series in the small quantities x_1 and y_1. We then obtain an expansion of the potential φ in multipoles of the charge. The first term,

$$\varphi^{\rm I} = QS_0, \quad Q = \int\int \rho_{eff} dx_1 dy_1, \quad S_0(x; \epsilon, y) = \sum_0^\infty \frac{\lambda_n(x)\lambda_n(\epsilon)}{\sqrt{n+2}} e^{-|y|\sqrt{n+2}}, \qquad (6)$$

will be nonzero if the "total charge" Q is nonzero. Solutions with $Q \neq 0$, however, describe perturbations which are periodic over the length of the pinch,[5] for which we need periodic "nucleating centers." If we assume that these nucleating centers could not arise under the conditions prevailing in space, we should regard solutions with $Q \neq 0$ as *unrealizable*! Solutions with $Q = 0$, on the other hand, are not periodic; they are instead local. The simplest of them is the solution $\varphi^{\rm II} = DS'_{0\varepsilon}$ with a "dipole moment" $D = \int x_1 \rho_{\rm eff} dx_1 dy_1$. Since $\lambda_0 = 1$ and $d\lambda_0/d\varepsilon = 0$, the sum $\partial S_0/\partial\varepsilon$ in this solution begins with the term with the factor of $\exp(-|y|\sqrt{3})$, which gives us the "cosmic spectrum" $\sim E^{-(1+\sqrt{3})}$, as we will show below.

To determine the spectrum, we note that the number of particles over a distance dz is $dN = \pi a^2 n\gamma dz = F(u)du$. Using the notation $N_0 = \pi a_0^2 n_0$ for brevity, we find the particle distribution function from the equations above:

$$F = \left(\frac{\partial N}{\partial u}\right)_t = N_0 \frac{\epsilon}{\gamma} e^{-x}[(\varphi - \psi'_y)^2 + x(\psi - \varphi'_y)^2][\varphi - \psi'_y + x(\psi - \varphi'_y)\tanh y]^{-1}. \qquad (7)$$

This expression gives us the spectrum at an arbitrary time. A typical perturbation, however, is a bulge between two necks. These necks rupture at $t = 0$, and at the same time the bulge converts into a pancake, which contains the particles squeezed out of the necks. In this pancake we have $\rho_* \to \infty$ and $x \to 0$, and since at small $x \ll 1$ we have

$$\varphi(x,y) = \varphi_0(y) + x\varphi_1(y) + x^2\varphi_2(y) + \ldots, \quad \psi(x,y) = \frac{x}{2}\varphi'_0(y) + \frac{x^2}{3}\varphi'_1(y) + \ldots, \qquad (8)$$

the distribution function in the limits $t \to 0$, $x \to 0$ is $F_0 = N_0 \varepsilon \varphi_0(y)/\gamma$. In particular, for the "dipole solution" we find

$$\varphi_0(y) = -|D| \sum_0^\infty \sqrt{\frac{k+2}{k+3}} e^{-|y|\sqrt{k+3}} \lambda'_{k+1}(\epsilon), \quad F_0(y \gg 1) = C\gamma^{-(1+\sqrt{3})}. \qquad (9)$$

This asymptotic expression is valid for essentially all local perturbations. It is actually the same as the observed spectrum of galactic cosmic rays. This agreement makes our "pinch mechanism" for the production of galactic cosmic rays extremely plausible.

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